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Flow of a Weakly Conducting Fluid in a Channel Filled with a Darcy–Brinkman–Forchheimer Porous Medium

B. Q. Zhao · A. Pantokratoras · T. G. Fang · S. J. Liao

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Abstract We investigate in this article, the fully developed flow in a fluid-saturated channel filled with a Darcy–Brinkman–Forchheimer porous medium, which is conducted with an electrically varying parallel Lorentz force. The Lorentz force varies exponentially in the vertical direction due to low fluid electrical conductivity and the special arrangement of the magnetic and electric fields at the lower plate. With the homotopy analysis method (HAM), a particularly effective technique in solving nonlinear problems, analytical approximation series solutions with high accuracy are derived for fluid velocity and the results are illustrated in form of figures. All these flows are new and are presented for the first time in the literature.

Keywords Porous medium · Couette flow · Poiseuille flow · Lorentz force · Analytical solution · HAM

1 Introduction

The problem of fully developed forced convection in a parallel plate channel filled with a fluid-saturated porous medium has attracted considerable attention in the past. It is the counterpart of a pure fluid flow between parallel plates named as Poiseuille or Couette flow.

B. Q. Zhao · S. J. Liao (✉)
State Key Lab of Ocean Engineering, Shanghai Jiao Tong University, Shanghai 200030, China
e-mail: sjliao@sjtu.edu.cn

B. Q. Zhao
e-mail: bingqianzhao@sjtu.edu.cn

A. Pantokratoras
School of Engineering, Democritus University of Thrace, 67100, Xanthi, Greece
e-mail: apantokr@civil.duth.gr

T. G. Fang
Department of Mechanical and Aerospace Engineering, North Carolina State University, 3182 Broughton Hall-Campus, 2601 Stinson Drive, Box 7910, Raleigh, NC 27695, USA
e-mail: tfang2@ncsu.edu

The study of convection through a porous medium channel was initiated by Kaviany (1985) who presented an analytical solution based on the Brinkman–Darcy flow model. A complete review of the studies was not available until 1996 given by Nield et al. (1996), and included the articles by Poulikakos and Renken (1987), Renken and Poulikakos (1988), Nakayama et al. (1988), and Vafai and Kim (1989). Nield et al. (1996) studied the fully developed forced convection in a porous medium channel bounded by parallel plates with uniform heat flux or with constant and equal temperatures. They used the Brinkman–Darcy–Forchheimer model and obtained an "exact solution" for this problem. Kuznetsov (1998) studied analytically the heat transfer in a Couette flow through a porous medium utilizing the Brinkman–Forchheimer-extended Darcy model. Haji-Sheikh (2004) studied the heat transfer between parallel plates using the Brinkman–Darcy model. Nield et al. (2004) investigated the fully developed forced convection in a channel using the Brinkman–Darcy model, taking into account the viscous dissipation and flow work. Hooman (2008) investigated the Poiseuille flow through a porous medium on the basis of a Brinkman–Forchheimer model using an asymptotic expansion method. In 1961, Gailitis and Lielausis (1961) introduced the idea of using a Lorentz force to control the flow of an electrically conducting fluid over a flat plate. This was achieved by applying an external electromagnetic field (see Fig. 1) with a stripwise arrangement of flush-mounted electrodes and permanent magnets of alternating polarity and magnetization. The Lorentz force, which acts parallel to the plate, can either assist or oppose the flow. This idea was later abandoned and only recently attracted new attention (Henoch and Stace 1995; Crawford and Karniadakis 1997; O'Sullivan and Biringen 1998; Berger et al. 2000; Kim and Lee 2000; Du and Karniadakis 2000; Breuer et al. 2004; Lee and Sung 2005; Spong et al. 2005). In addition, in the past years much investigation on flow control using the Lorentz force was conducted at the Rossendorf Institute and at the Institute for Aerospace Engineering in Dresden, Germany (Weier et al. 1998; Posdziech and Grundmann 2001; Weier et al. 2003; Weier and Gerbeth 2004; Mutschke et al. 2006; Albrecht et al. 2006; Shatrov and Gerbeth 2007). Pantokratoras (2007) presented an exact analytical solutions of the Poiseuille and the Couette flows for a clear fluid under the action of a parallel Lorentz force. Pantokratoras and Fang (2009) investigated the Poiseuille and Couette flows in a Darcy–Brinkman porous medium under the effect of a parallel Lorentz force and presented exact analytical solution. Magyari (2009) presented a new analytical solutions for a case not treated in the above work Pantokratoras and Fang (2009). In this article, we will extend the previous work Pantokratoras and Fang (2009) to the case with a Darcy–Brinkman–Forchheimer porous medium. The present problem, in contrast to the previous one, does not accept analytical solutions and will be treated by the HAM method. We will study the flow between two parallel, infinite plates filled with a porous medium with a Lorentz force created at the lower plate according to the arrangement shown in Fig. 1.

The so-called homotopy analysis method, which was first proposed by Liao (2003a,b, 2006); Liao and Tan (2007), has evolved into an excellent and unique system for solving nonlinear problems. Based on homotopy, which is a basic concept in topology, an analytic method, namely the homotopy analysis method (HAM), was proposed and then widely applied to solve strongly nonlinear problems in science, engineering, and finance (Liao and Pop 2004; Yamashita et al. 2007; Bouremel 2007; Abbasbandy 2007a,b; Hayat and Sajid 2007a,b; Allan 2007; Sajid et al. 2006; Zhu 2006a,b; Liao 2005; Liao and Magyari 2006). Unlike perturbation techniques, the homotopy analysis method is independent of any small/large physical parameters. Besides, different from perturbation and traditional non-perturbation methods, it provides a simple way to ensure the convergence of solution series so that one can always get accurate enough approximations even for strongly nonlinear problems. Furthermore, unlike all other analytic techniques, the homotopy analysis method

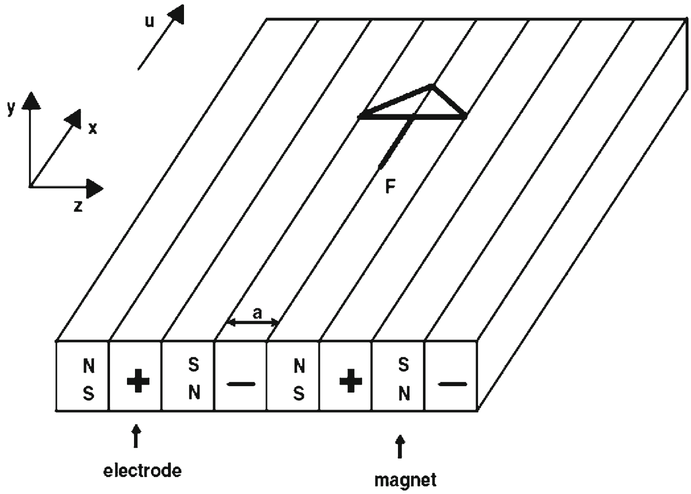

Fig. 1 The Riga plate with alternating arrangement of electrodes and permanent magnets

provides great freedom to choose the so-called auxiliary linear operator so that one can approximate a nonlinear problem more effectively by means of better base functions. This kind of freedom is so significant that the second-order nonlinear two-dimensional Gelfand equation can be solved even by means of a fourth-order auxiliary linear operator, as shown in Liao and Tan (2007). Especially, by means of the homotopy analysis method, a few new solutions of some nonlinear problems were found (Liao 2005; Liao and Magyari 2006), which were neglected by all other analytic methods and even by numerical techniques. Furthermore, the homotopy analysis method has been applied to solve some nonlinear partial differential equations (Liao 2006), such as unsteady similarity boundary-layer flows, Black-Scholes type equation in finance for American put option (Zhu 2006a,b), and so on. These previous studies provide us a good background to apply the HAM to solve nonlinear ODE.

In this article, we present an analytic approach to solve a nonlinear equation of both Couette and Poiseuille flows between two plates with a Darcy–Brinkman–Forchheimer porous medium. In §2, the mathematical model and basic ideas of the HAM approach are described. In §3, two kinds of flows are investigated and the accurate series solutions are obtained, which are convergent and valid in a large spatial regions. Discussions and conclusions are given in §4.

2 Mathematical Description

2.1 The Mathematical Model

The Couette and Poiseuille flows between two parallel plates, in which the lower plate is a Riga plate, have been solved analytically by Pantokratoras (2007). In the present article, we investigate the case where the space between the plates is filled with a Darcy–Brinkman–Forchheimer porous medium. The flow between the plates is assumed fully developed. This means that the horizontal velocity is a function of y only. The governing equations of these problems are:

$$-\frac{\mathrm{d}p}{\mathrm{d}x}+\mu_{\mathrm{eff}}\frac{\mathrm{d}^2u}{\mathrm{d}y^2}+\frac{\pi j_0 M_0}{8}\exp\left(\frac{-\pi}{a}y\right)-\frac{\mu}{K}u-\frac{c_{\mathrm{F}}\rho}{K^{1/2}}u^2=0 \tag{1}$$

where x is the horizontal coordinate, y is the vertical coordinate, u is the fluid velocity along the plates, K is the porous medium permeability, $\mathrm{d}p/\mathrm{d}x$ is the pressure gradient, j_0 $\left(Amp/m^2\right)$ is the applied current density in the electrodes, M_0 is the magnetization of the permanent magnets, a is the width of magnets and electrodes, μ is the fluid dynamic viscosity and μ_{eff} is the effective viscosity, c_{F} is the Forchheimer coefficient. The most recent information concerning the effective viscosity was given by Nield and Bejan (2006). The effective viscosity is not fully understood until now and its value may be 10 times higher than the fluid viscosity. The boundary conditions are:

$$\begin{aligned} &u(0)=0, u(h)=0, &&\text{for the Poiseuille flow}\\ &u(0)=0, u(h)=u_{\mathrm{w}}, &&\text{for the Couette flow} \end{aligned} \tag{2}$$

where h is the distance between the plates, u_{w} is the velocity of the upper plate for the Couette flow. Equation 1 can be non-dimensionalized by defining $U=u/u_{\mathrm{r}}$, and $Y=y/h$ as

$$-\frac{1}{u_{\mathrm{r}}}\frac{\mathrm{d}p}{\mathrm{d}x}+\frac{\mu_{\mathrm{eff}}}{h^2}\frac{\mathrm{d}^2U}{\mathrm{d}Y^2}+\frac{\pi j_0 M_0}{8u_{\mathrm{r}}}\exp\left(-\frac{\pi h}{a}Y\right)-\frac{\mu}{K}U-\frac{c_{\mathrm{f}}\rho u_{\mathrm{r}}}{K^{1/2}}U^2=0 \tag{3}$$

It can be converted to

$$A+\frac{\mathrm{d}^2U}{\mathrm{d}Y^2}+Q\exp(-BY)-DaU-FU^2=0 \tag{4}$$

where

$$A=-\frac{h^2}{\mu_{\mathrm{eff}}u_{\mathrm{r}}}\frac{\mathrm{d}p}{\mathrm{d}x},\ Q=\frac{\pi j_0 M_0}{8u_{\mathrm{r}}}\frac{h^2}{\mu_{\mathrm{eff}}},\ B=\frac{\pi h}{a},\ Da=\frac{\mu h^2}{K\mu_{\mathrm{eff}}},\text{ and } F=\frac{c_{\mathrm{f}}\rho u_{\mathrm{r}}}{K^{1/2}}\frac{h^2}{\mu_{\mathrm{eff}}}. \tag{5}$$

where A, Q, B, Da, and F are all constants that do not depend on y, the parameter Q expresses the balance between the electromagnetic forces and viscous forces and is equivalent to square of the classical Hartmann number or to Chandrasekhar number while Da is the Darcy number used in the theory of porous media. Parameter F is the Forchheimer number, which shows the non-Darcian effects in the porous medium. The physical meaning of F is to show how the porous medium deviates from the conventional Darcy medium. The range of F is from 0 to infinity. u_{r} is a reference velocity.

For the Poiseuille flow, we can define $A=1$ by setting $u_{\mathrm{r}}=-\frac{h^2}{\mu_{\mathrm{eff}}}\frac{\mathrm{d}p}{\mathrm{d}x}$, and the boundary conditions become

$$U(0)=U(1)=0 \tag{6}$$

For the Couette flow, $u_{\mathrm{r}}=u_{\mathrm{w}}$. The boundary conditions become

$$U(0)=0, U(1)=1. \tag{7}$$

In the Couette flow, we can still keep the pressure gradient as a combined Couette–Poiseuille flow with $A\neq 0$. For $A=0$, it will be a pure Couette flow.

As has been mentioned before, when $F=0$, it becomes the flow between two parallel plates in which the lower plate is a Riga plate, which has been solved analytically by Pantokratoras and Fang (2009). The exact solution is:

$$U(Y)=\frac{Q\exp(-By)}{Da-B^2}+\frac{A}{Da}+\exp\left(\sqrt{Da}Y\right)C_1+\exp\left(-\sqrt{Da}Y\right)C_2 \tag{8}$$

where C_1 and C_2 are integration constants, which can be determined by the boundary conditions. For the combined Couette–Poiseuille flow:

$$
\begin{aligned}
C_1 &= -\frac{Q}{Da - B^2} - \frac{A}{Da} - \frac{1 + \frac{Q\left(\exp(\sqrt{Da}) - \exp(-B)\right)}{Da - B^2} + \frac{A\left(\exp\left(\sqrt{Da}\right) - 1\right)}{Da}}{\exp\left(-\sqrt{Da}\right) - \exp\left(\sqrt{Da}\right)} \\
C_2 &= \frac{1 + \frac{Q\left(\exp\left(\sqrt{Da}\right) - \exp(-B)\right)}{Da - B^2} + \frac{A\left(\exp\left(\sqrt{Da}\right) - 1\right)}{Da}}{\exp\left(-\sqrt{Da}\right) - \exp\left(\sqrt{Da}\right)}
\end{aligned}
\tag{9}
$$

and the Poiseuille flow:

$$
\begin{aligned}
C_1 &= -\frac{Q}{Da - B^2} - \frac{1}{Da} - \frac{\frac{Q\left(\exp\left(\sqrt{Da}\right) - \exp(-B)\right.}{Da - B^2} + \frac{\exp\left(\sqrt{Da}\right) - 1}{Da}}{\exp\left(-\sqrt{Da}\right) - \exp\left(\sqrt{Da}\right)} \\
C_2 &= \frac{\frac{Q\left(\exp\left(\sqrt{Da}\right) - \exp(-B)\right)}{Da - B^2} + \frac{\exp\left(\sqrt{Da}\right) - 1}{Da}}{\exp\left(-\sqrt{Da}\right) - \exp\left(\sqrt{Da}\right)}
\end{aligned}
\tag{10}
$$

2.2 The Analytical Approach Based on HAM

Due to the boundary conditions and the known exact solutions for $F = 0$, $u(y)$ for a positive F can be expressed by a set of base functions:

$$\left\{ y^m \exp(-nBy) \mid m \geqslant 0, n \geqslant 0 \right\} \tag{11}$$

in the following form:

$$u(y) = \sum_{m=0}^{+\infty} \sum_{n=0}^{+\infty} a_{m,n} y^m \exp(-nBy) \tag{12}$$

where $a_{m,n}$ is a coefficient, then according to the above expression and the boundary conditions, for simplicity we can choose the initial solution as :

$$u_0(y) = \begin{cases} 0, & \text{for the Poiseuille flow,} \\ y, & \text{for the Couette flow.} \end{cases} \tag{13}$$

Also, because of the governing equation 4 and solution expression equation 11, we define our auxiliary linear operator as:

$$\mathcal{L}[\phi(y;q)] = \frac{\partial^2 \phi(y;q)}{\partial y^2} \tag{14}$$

Note that the linear operator $\mathcal{L}$ has such a property:

$$\mathcal{L}[C_1 y + C_2] = 0. \tag{15}$$

Besides, let c_0, also called convergence-controlling parameter, denote a non-zero auxiliary parameter, $q \in [0, 1]$ an embedding parameter, respectively. According to Eq. 4, we define a nonlinear operator as:

$$\mathcal{N}[\phi(y;q)] = \frac{\partial^2\phi(y;q)}{\partial y^2} - Da\phi(y;q) - F\phi(y;q)^2 + Q\exp(-By) + A. \tag{16}$$

Then, we can construct a zero-order deformation equation:

$$(1-q)\mathcal{L}[\phi(y;q) - u_0] = qc_0\mathcal{N}[\phi(y;q)] \tag{17}$$

subject to the boundary conditions:

$$\begin{aligned} &\phi(0;q) = 0, \phi(1;q) = 0 \quad \text{for the Poiseuille flow} \\ &\phi(0;q) = 0, \phi(1;q) = 1 \quad \text{for the Couette flow} \end{aligned} \tag{18}$$

When $q = 0$, it is straightforward to show that the solution is

$$\phi(y;0) = u_0(y) \tag{19}$$

when $q = 1$, since $c_0 \neq 0$, then the zero-order deformation equation (17) is equivalent to Eq. 4, provided:

$$\phi(y;1) = u(y) \tag{20}$$

Thus, as q increases from 0 to 1, $\phi(y;q)$ varies from the initial guess $u_0(y)$ defined by Eq. 13, to the exact solution $u(y)$ governed by Eq. 4. This kind of continuous variation is called deformation in topology.

By using the Taylor's theorem, it is straightforward to expand $\phi(y;q)$ in a power series of the embedding parameter q as follows:

$$\phi(y;q) = u_0 + \sum_{m=1}^{+\infty} u_m q^m \tag{21}$$

where

$$u_m(y) = \frac{1}{m!}\frac{\partial^m\phi(y;q)}{\partial q^m}\bigg|_{q=0.} \tag{22}$$

Assuming that all of them are correctly chosen so that the series solution equation 21 converges at $q = 1$, we then have from Eq. 20 that

$$u(y) = u_0(y) + \sum_{m=1}^{+\infty} u_m(y) \tag{23}$$

Define the vectors:

$$\vec{u}_m = \{u_0, u_1, u_2, \cdots, u_m\}$$

Differentiating the zero-order deformation equation m times with respect to q and then dividing them by $m!$ and then finally setting $q = 0$, we have the mth-order deformation equation:

$$\mathcal{L}\left[u_m - \chi_m u_{m-1}\right] = c_0 R_m(y) \tag{24}$$

for both the Poiseuille and the Couette flows subject to the boundary conditions:

$$u_m(0) = 0, u_m(1) = 0 \tag{25}$$

where

$$R_m(y) = \frac{1}{(m-1)!}\left\{\frac{\partial^{m-1}\mathcal{N}[\phi(y;q)]}{\partial q^{m-1}}\right\}\bigg|_{q=0}$$
$$= u''_{m-1} - F\sum_{j=0}^{m-1-j} u_j u_{m-1-j} - Dau_{m-1} + (1-\chi_m)\left[A + Q\exp(-By)\right] \quad (26)$$

and

$$\chi_m = \begin{cases} 0, & \text{when } m \leqslant 1, \\ 1, & \text{when } m > 1. \end{cases} \quad (27)$$

Let $u_m^*(y)$ denote a particular solution of Eq. 24. From Eq. 14, its general solution reads

$$u_m(y) = u_m^*(y) + C_1 y + C_2, \quad (28)$$

where the coefficients C_1, C_2 are determined by the boundary conditions equation (25). In this way, it is easy to solve the linear equations (24–25) successively.

At this point, we have successfully transformed the non-linear equation (4) into a series of linear equations. By using a symbolic computation software such as Mathematica, Maple, MatLab, and so on, it is not difficult to get the series solutions.

3 Results

As has been mentioned before, by choosing a proper solution expression, linear operator $\mathcal{L}$, convergence-controlling parameter c_0, and the HAM can always give a family of convergent series solutions containing c_0. That is to say, with parameter c_0 varying in such a region called R_c, the series solutions converge to the same value, reflecting in the figure a length of horizontal line. What is more, we can choose the optimum c_0 that gives the fastest convergent series from such a valid region R_c by checking the exact residual error presented in the form:

$$\Delta = \int_0^\infty \left(\mathcal{N}\left[\sum_{i=0}^{m} u_i(y)\right]\right)^2 \mathrm{d}y \quad (29)$$

or the so-called averaged residual error defined by

$$E_m = \frac{1}{N}\sum_{j=0}^{N}\left[\mathcal{N}\left(\sum_{n=0}^{m} F_n(j\Delta x)\right)\right]^2 \quad (30)$$

Equation 30 is used when more CPU time is required for calculating the exact residual error Δ. For example, for the Poiseuille flow when the parameter $F = 1$, we portrayed in Fig. 2 the curves of averaged residual error E_m versus c_0 at each given order and demonstrated in Table 1 the exact value of the averaged residual error. We can see from Fig. 2 that: the curves at any given order intersect with each other at two different points, $c_0 = -0.5, c_0 = 0$, respectively, which corresponds to the valid region R_c. There also exists an optimum point for c_0 at which the homotopy series equation (23) gives the fastest series solutions. In this case, we can see from Fig. 2, the optimum point is about -0.4. In the same way, $c_0 = -2/5$ are chosen for all of the following cases concerned. We demonstrate the comparison between the HAM results with exact solution when $F = 0$ for the Poiseuille flow in Fig. 3 and for the

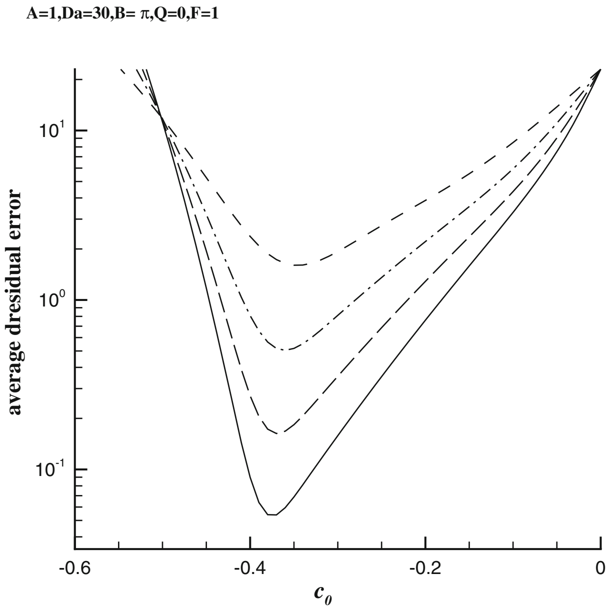

Fig. 2 Exact residual error Δ versus c_0 for the Poiseuille flow when parameter $F = 1$. *Dashed line* second-order approximation; *Dash-Dotted line* third-order approximation; *Dotted line* fourth-order approximation; *solid line* fifth-order approximation

Table 1 Comparison of error for various order HAM approximations

*m*th order approximations	Averaged residual error	Value of $u(1/2)$
5	1.43×10^{-1}	0.10771
10	5.55×10^{-4}	0.09831
15	2.76×10^{-6}	0.09866
20	1.92×10^{-8}	0.09867

Couette flow in Fig. 4, respectively. As is portrayed in both figures, all of the cases concerned match very well with the exact solution for the 20-order of HAM series results, proving the validity of HAM.

In addtion, the present problem we are trying to solve contains a parameter F. Parameter F is also called the Forchheimer number, which shows the non-Darcian effects in the porous medium. For conventional Darcian medium, the effects of porous medium is only modelled as $-Da * U$, but for the non-Darcian media, the effect of porous medium is modelled as $-Da * U - F * U^2$. There is a nonliner term in the equation. Especially with varying values of F, Eq. 4 can evolve into a very strong nonlinear problem. With HAM, however, various values of c_0 is chosen to assure the convergence of the series solution of Eq. 1. For example, we choose $c_0 = -1/4, -3/20, -1/25$ when $F = 10, 100, 1000$, respectively. The results are plotted in Fig. 5 for the Poiseuille flow and in Fig. 6 for the Couette flow.

The physical meaning of F is to show how the porous medium deviates from a conventional Darcian medium. The range of F is from 0 to infinity. As for the effects of F on the velocity profiles, since there are more damping forces due to the non-linear Forchheimer

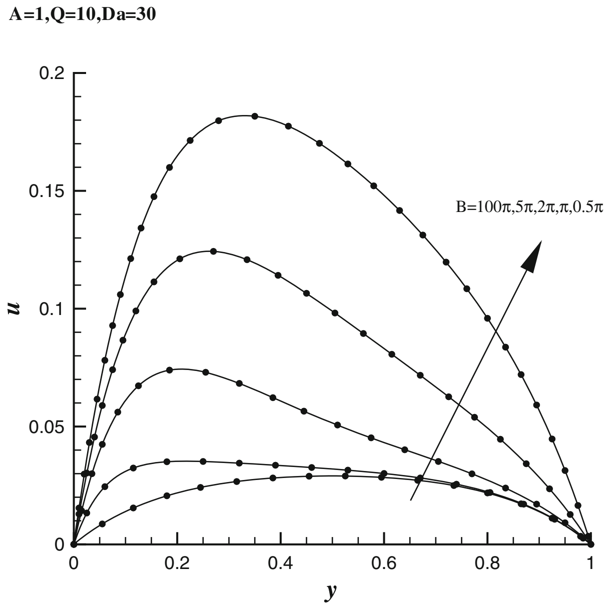

Fig. 3 Velocity profiles comparison for the Poiseuille flow between exact solution and HAM when $A = 1$, $Da = 30$, $Q = 10$ and different values of B; *solid line* HAM results; *circles* exact results

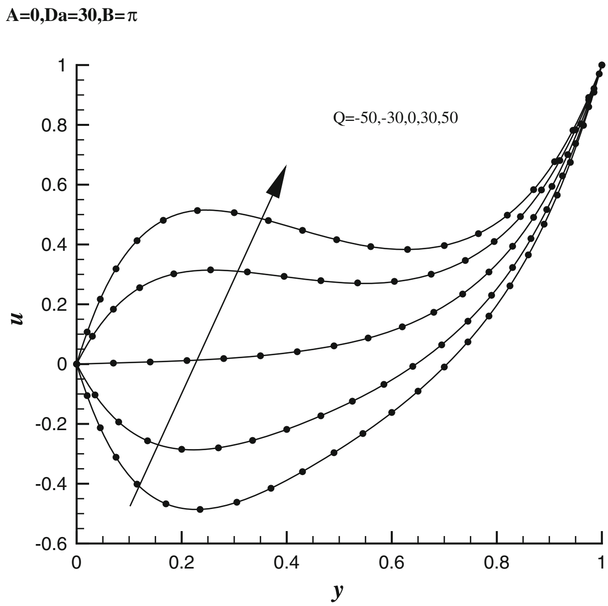

Fig. 4 Velocity profiles comparison for the Couette flow between exact solution and HAM results when $A = 0$, $Da = 30$, $B = \pi$ and different values of Q; *solid line* HAM results; *circles* exact results

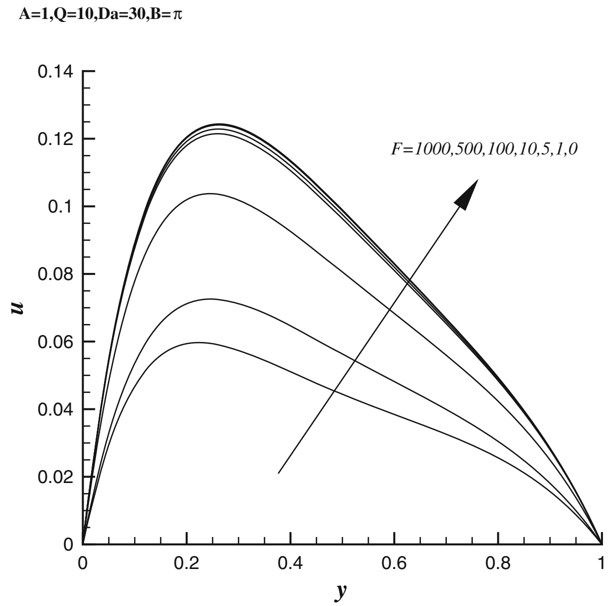

Fig. 5 Velocity profiles for the Poiseuille flow when $A = 1$, $Q = 10$, $Da = 30$, $B = \pi$ and different values of the Forchheimer number F

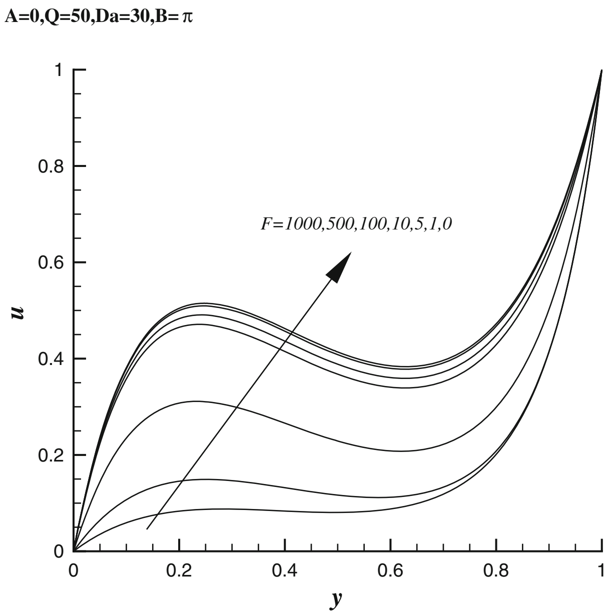

Fig. 6 Velocity profiles for the Couette flow when $A = 0$, $Q = 50$, $Da = 30$, $B = \pi$ and different values of the Forchheimer number F

effects applied on the fluid for higher F medium, the fluid velocity decays faster with the increase of the distance from the wall, as is illustrated in Figs. 5 and 6.

4 Conclusion

In this article, the problem of the Poiseuille and Couette–Poiseulle flows between two parallel plates filled with a Darcy–Brinkman–Forchheimer porous medium under the influence of a horizontal Lorentz force has been investigated. Approximate analytical solutions have been given for velocity and velocity profiles have been presented for different values of the Forchhimer number F, when other parameters like Da, Q and B are given to the flow. The influence of these parameters has been explained physically.

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