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Observations of highly localized oscillons with multiple crests and troughs

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Two types of stable, highly localized Faraday resonant standing waves with multiple crests and troughs are observed in an ethanol-water solution partly filled in a Hele-Shaw cell vertically oscillated with a single frequency. Systematical experiments are performed to investigate the properties of these oscillons. It is found that the wave height of these oscillons is independent of fluid depth from 1 to 5 cm. In particular, some experiments are performed to indicate the high localization of the oscillons, which suggests that these oscillons may be regarded as a combination of the two elementary oscillons discovered by Rajchenbach *et al.* [Phys. Rev. Lett. **107**, 024502 (2011)], for instance, $(2,3) = (1,1) + (1,2)$, where (m,n) denotes an oscillon with m crests and n troughs. So, our experiments also reveal an elegant "arithmetic" of these oscillons. These experimental phenomena are helpful to deepen and enrich our understanding about Faraday waves.

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Localized Faraday resonant standing waves sloshing between the wall have been observed by Wu *et al.* [1] in the vertically oscillated water partly filled in a three-dimensional trough. The measurements of the corresponding velocity field have been obtained by particle image velocimetry and an extra streaming velocity has been found in [2]. Recently, two new standing solitary waves with odd and even symmetries have been found in a Hele-Shaw cell partly filled with distilled water [3] when the cell is under vertically periodic vibrations. Since the distance between the walls of the Hele-Shaw cell is quite small, there is no sloshing property for the two waves. Other kinds of excitations in a layer of suspension have been observed as well in experiments. Both harmonic and subharmonic oscillons and propagating solitons have been reported [4–6]. The transition from oscillons to propagating solitons has been investigated as well [7]. The mobility of the oscillon can be affected by the amount of protein in the suspension, and waves become well organized by increasing the concentration of protein [8]. Instead, the ordered structure of the waves will break down if the amplitude of the vibration grows [9]. Interesting walkers and droplets have been observed as well in a similar situation when the acceleration reaches a critical threshold [10]. In the theoretical research, stationary spatially localized structures have been described in detail in a well-defined region in parameter space. The corresponding stability of the states with respect to two-dimensional perturbations has been examined as well [11].

In this Rapid Communication, we report the observations of some standing oscillons with multiple crests and troughs in a Hele-Shaw cell partly filled with an ethanol-water solution. It should be mentioned that in a granular medium Umbanhowar *et al.* [12] have observed the three-dimensional oscillons "dipole" and "polymeric." Although these "dipole" and "polymeric" oscillons have similar behavior to some of the localized waves observed in this Rapid Communication in the transverse view, the oscillons with multiple crests and troughs are reported here for water waves in a Hele-Shaw geometry. In particular, they are highly localized, and can be regarded as a combination of the two elementary oscillons (with odd and even symmetry) discovered by Rajchenbach *et al.* [3].

In our experiment, a thin layer of ethanol-water solution in a Hele-Shaw cell is considered. The cell is made of polymethyl methacrylate (PMMA) and is 30 cm long and 1.7 mm in breadth (the same size as [3]). It is mounted on a shaker vertically oscillated with a displacement $z = A\sin(2\pi f t)$, where the constants A and f denote the amplitude and frequency of the oscillation. The frequency and oscillation format are determined by the input signal, which is already an electrical signal and is visible. Based on these input settings, the shaker can vibrate at the determined frequency and in the determined oscillation format. Then through the conditioning amplifier, the information of the acceleration is displayed on an oscilloscope. The acceleration A has the following relationship with the reading of the oscilloscope E: $A = 0.1414E$. For example, when the reading of the oscilloscope is $E = 145$ mV, it corresponds to the acceleration $A = 20.503$ m/s^2. In the practical operation, we regulate the reading of the oscilloscope to determine the value of acceleration. It should be mentioned that since both the shaker and the wave form of the oscilloscope are stable, the acceleration is accurate through the whole experiment. The temperature is confined at about 20°. Considering the volatility, the ethanol-water solution is changed every 10 minutes so as to guarantee the constant concentration. Using a high-speed camera (250 fps) positioned perpendicular to the cell, the images are acquired once the oscillons have stable and distinct

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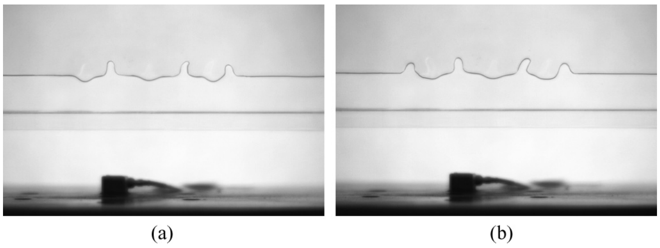


FIG. 1. The oscillons (3,3) and (4,3). The new oscillons observed in 15% ethanol-water solution with a fluid depth of 2 cm, an oscillation frequency of 18 Hz, and an acceleration amplitude of 20.503 $\mathrm{m/s^2}$. (a) and (b) represent the typical images of the oscillons (3,3) and (4,3), respectively. The corresponding supplementary movies [13] are available online.

forms with a typical lifetime of about 1000 oscillation periods. After around 1000 periods, the amplitude of the oscillons will become smaller and smaller and disappear at last. This may result from the viscosity.

All experiments are performed with a 15% ethanol-water solution unless noted otherwise. First, the experiments were performed in the solution with a fluid depth of 2 cm, an oscillation frequency of 18 Hz, and an acceleration amplitude of 20.503 $\mathrm{m/s^2}$. We disturbed the free surface by an extra probe as the initial, local excitation. Dependent upon the way we disturbed the free surface, stable subharmonic standing resonant waves with different shapes are observed. When the disturbed region of free surface is short (but strong enough to excite the resonance), the same oscillons with the even and odd symmetry reported in [3] are observed. It is very interesting that new oscillons with multiple crests and troughs are observed when the disturbed region of free surface is enlarged, or an additional disturbance is given near the free surface of an observed oscillon. They are highly localized, as shown in Fig. 1, and have stable and distinct forms with a typical lifetime of 1000 oscillation periods. Let (m,n) denote such an oscillon with m crests and n troughs at time t. The oscillon (m,n) at t becomes (n,m) at $t+T$ where T denotes the oscillation period of the shaker, namely, that the m crests and n troughs at t become the corresponding m troughs and n crests without changing the location after one oscillation period T of the shaker. So, (m,n) and (n,m) denote the same oscillon. These oscillons can be divided into two types. One corresponds to $m=n$, the other to $|m-n|=1$. Note that $(1,1)$ and $(1,2)$ correspond to the elementary oscillons with the odd and even symmetry reported in [3]. Figures 1(a) and 1(b) present the typical images of the oscillons (3,3) and (4,3), when their crests reach the maximum positions. At the acceleration amplitude of 20.503 $\mathrm{m/s^2}$, 12 oscillons, say, (1,1), (1,2), (2,2), (2,3), (3,3), (3,4), (4,4), (4,5), (5,5), (5,6), (6,6) and (6,7), are observed. It seems that localized oscillons with more crests and troughs can be observed, if the cell is long enough. Although Umbanhowar *et al.* [12] have observed the three-dimensional oscillons "dipole" and "polymeric" in a granular medium which, in the transverse view, have similar behavior to some of the localized waves observed in this Rapid Communication, these observed localized standing waves with multiple crests and troughs are totally new in a fluid.

Secondly, we fixed the frequency at 18 Hz and the acceleration amplitude at 20.503 $\mathrm{m/s^2}$, respectively, but changed the fluid depth from 1 to 5 cm so as to investigate the influence of fluid depth on the oscillons. Without loss of generality, the (2,2) oscillon was studied. The adjacent crest-to-trough height was recorded when the crests and troughs approached their corresponding highest and lowest positions, respectively. Two values of the adjacent crest-to-trough height were measured when the two crests of the oscillon (2,2) approached their highest position. After one oscillation period of the shaker, another two values were gained. Averaging these four values, the so-called averaged wave height of the oscillon (2,2) was obtained. Considering the uncertainty of the initial disturbance of free surface, five independent experiments were done under the same physical situation (in fact, all experimental results reported in this Rapid Communication were obtained in this way), so that five values of the averaged wave height of (2,2) are given. It is found that the five wave heights have an averaged value 10.45 ± 0.48 mm (average $\pm$ standard deviation). Thus, it is reasonable to use the averaged value 10.45 mm as the averaged wave height of the oscillon (2,2). Besides, it is found that the averaged wave heights in different fluid depths are almost the same, as shown in Fig. 2(a), where the depths are in the domain of 1 to 5 cm. This suggests that the averaged wave height of the oscillon (2,2) is almost independent of fluid depth. It is interesting that the oscillon is still localized and stable with the same wave height even in a fluid depth of 1 cm, corresponding to a highly nonlinear dynamic system. It should be mentioned that, according to our experiments, the fluid depth has indeed no effect on the wave height in the water depth from 1 to 5 cm. It is also found that fluid depth has almost no influence on other types of oscillons in the depth range from 1 to 5 cm, too. To the best of our knowledge, *neither* theoretical *nor* experimental works have been reported about this kind of independence of the oscillons on fluid depth.

To investigate the influence of the concentration of ethanol-water solution, the same experiments were performed at a frequency of 18 Hz, an acceleration amplitude of 20.503 $\mathrm{m/s^2}$, and a fluid depth of 2 cm. It is found that the stability of the oscillons is dependent upon the ethanol concentration. The oscillons cannot be observed because of large surface tension at the concentration of 5%. Besides, when the concentration is 20%, the oscillons become quite sensitive and easy to extend into the whole cell under perturbations. In addition, the ethanol-water solution with higher concentration is more volatile. However, for a small change in concentration of about 15%, the averaged amplitude does not change. For instance, when the concentration is changed from 15% to 10%, the same averaged wave height 10.45 ± 0.11 mm of the (2,2) oscillon is observed.

To investigate the influence of the acceleration amplitude of oscillation on the oscillons, experiments were performed at a frequency of 18 Hz and a fluid depth of 2 cm. Without loss of generality, the oscillon (2,2) was studied. It is found that the (2,2) oscillons are observed in a region of the acceleration amplitude [19.796,22.624], and their averaged wave heights increase almost linearly, as shown in Fig. 2(b). It is found that there exist the lower threshold $a_{\mathrm{low}} = 19.796$ $\mathrm{m/s^2}$ and the upper $a_{\mathrm{up}} = 22.624$ $\mathrm{m/s^2}$. When the acceleration amplitude is below the lower threshold a_{low}, the oscillon has a short lifetime

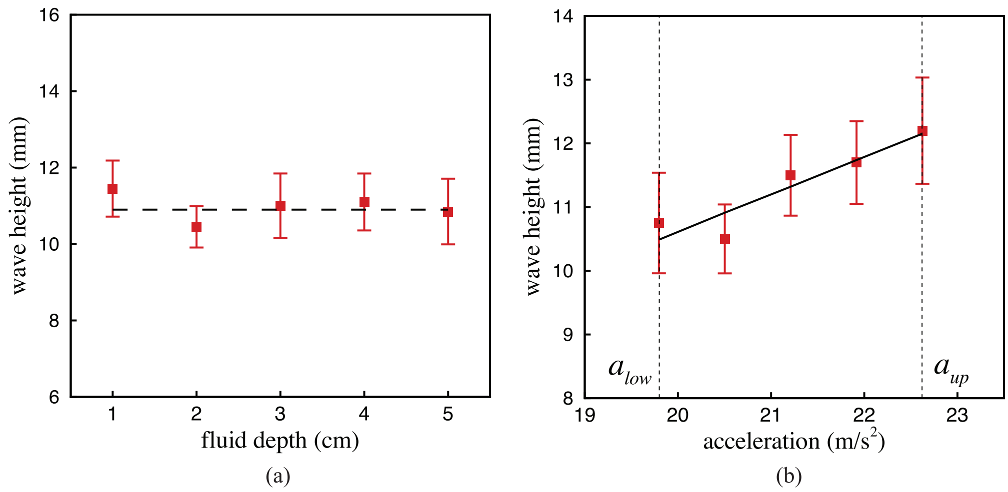


FIG. 2. (Color online) Averaged wave height versus fluid depth and acceleration amplitude. (a) Wave height of the (2,2) oscillon versus fluid depth, in a 15% ethanol-water solution at the frequency of 18 Hz and the acceleration amplitude of 20.503 m/s^2. The square symbols denote the mean values of the experimental results, with error bars for standard derivation. The dashed line represents the averaged value 10.45 mm. (b) Wave height of the (2,2) oscillon versus acceleration amplitude, in a 15% ethanol-water solution at a frequency of 18 Hz and fluid depth of 2 cm. Square symbols denote the mean value of the experimental results, with error bars for standard derivation. The solid line represents the optimal approximation obtained by the minimized linear least squares. a_{low} and a_{up} denote the lower and up thresholds of acceleration amplitude for the existence of oscillons, respectively.

and disappears after only a few periods of oscillations. Instead, when the acceleration amplitude is over the upper threshold a_{up}, the oscillon is not localized and expanded into the whole cell so that the surface becomes wavy. Note that a similar window of acceleration amplitude has been reported in [3] for the elementary oscillons (1,1) and (1,2), with a theoretical explanation. It is found that almost the same lower and up thresholds existed for other types of oscillons (m,n). Thus, our experiments indicate that such kind of window of the acceleration amplitude exists for the localized oscillon (m,n) in general, with the linear increase of averaged wave height with respect to the acceleration amplitude within this kind of existence-window.

Do such kind of windows exist in general? To answer this question, experiments were performed at the same fluid depth of 2 cm but different frequency of oscillation. It is found that a similar window of acceleration amplitude exists for frequency of oscillation from 12 to 20 Hz, as shown in Fig. 3. Both the lower and up thresholds of the existence window increase as the frequency enlarged, and various types of localized oscillons are observed within the windows. It is found that oscillons have larger size at lower frequency, and some oscillons cannot be observed below 12 Hz. Besides, stable oscillons with long enough lifetime cannot be observed at frequencies larger than 20 Hz.

To compare the averaged wave height of different types of oscillons (m,n), the experiments were performed at a frequency of 18 Hz, an acceleration amplitude of 20.503 m/s^2, and a fluid depth of 2 cm. It is a little surprising that the averaged wave heights of all types of observed oscillons are almost the same, say, 10.3 ± 0.19 mm, as shown in Fig. 4(a). In particular, the oscillons (1,1) and (1,2) have the same averaged wave height. It suggests that the different types of oscillons have the same averaged wave height for the fixed frequency and acceleration amplitude of oscillation at the same fluid depth. The adjacent crest-to-crest distance of these oscillons can be defined and measured in a similar way to give the so-called averaged crest-to-crest distance. It is found that the averaged crest-to-crest distance of these oscillons are irregular, with the maximum gap of 12.7 mm and the maximum standard deviation of 20.2%, as shown in Fig. 4(b). It should be mentioned that the symbols in Fig. 4(b) are obtained in the following ways. Take the oscillon (3,3) in Fig. 1(a) as an example. There are three crests. So there are two crest-to-crest

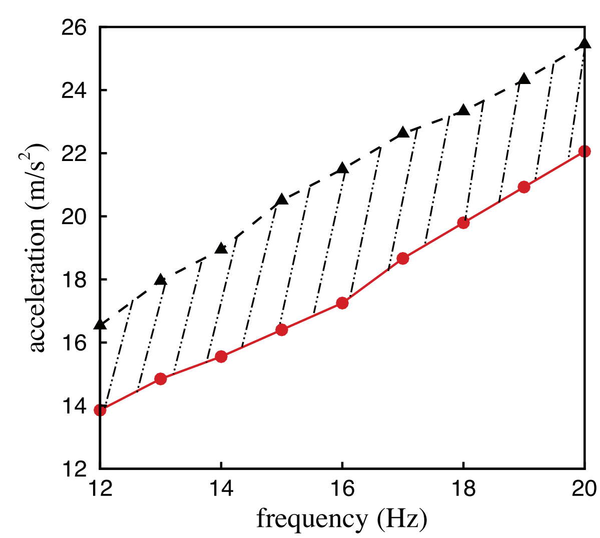


FIG. 3. (Color online) Existence window of oscillons. Experiments were performed in a 15% ethanol-water solution with a fluid depth of 2 cm. The solid line with circles denotes the lower threshold of acceleration amplitude, and the dashed line with triangles represents the upper one, respectively.

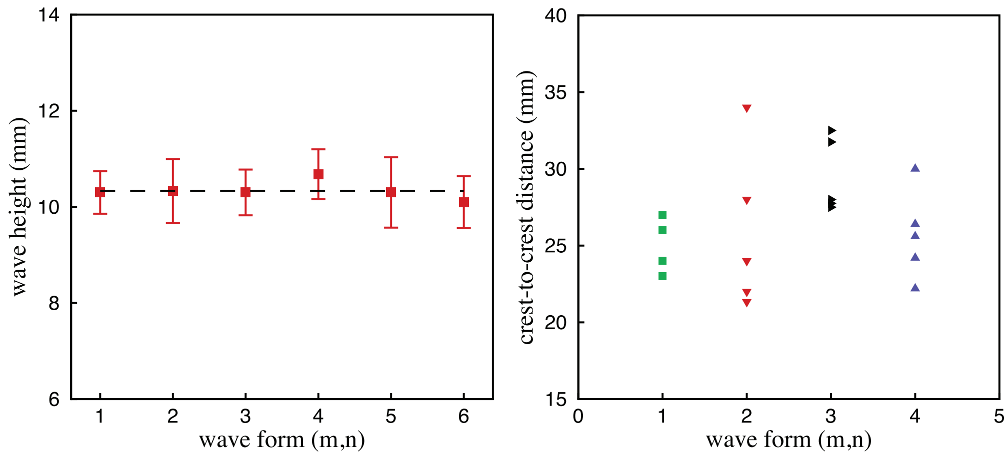


FIG. 4. (Color online) Wave height and crest-to-crest distance of oscillons. Experiments were performed in a 15% ethanol-water solution at a frequency of 18 Hz, an acceleration amplitude of 20.503 $\mathrm{m/s^2}$, and a fluid depth of 2 cm. (a) The averaged wave height of different types of oscillons. The integers 1 to 6 in the horizontal axis denote the oscillons of (1,1), (1,2), (2,2), (2,3), (3,3), and (3,4), respectively. Square symbols denote the mean value of the experimental results, with error bars for standard derivations. The dashed line represents their averaged value. (b) The averaged crest-to-crest distance of different types of oscillons. The integers 1 to 4 in the horizontal axis denote the oscillons of (2,2), (2,3), (3,3), and (3,4), respectively. Oscillons are created five times under the same physical situation. Each time when one oscillon is created, the crest-to-crest distance is measured once.

distances from the left to right. Next we can average these two crest-to-crest distances and obtain the averaged crest-to-crest distance for this oscillon (3,3). This averaged crest-to-crest distance corresponds to one symbol (let us denote this symbol as A) in Fig. 4(b). Next, another experiment with the same physical parameter was done to get another oscillon (3,3). The same measurements were done. Then we got another symbol (let us denote it as B) in Fig. 4(b). As the experiments were done again, we have symbols C, D, and E (five experiments were done for one oscillon). For these symbols, A,B,C,D,E, we did not average them further. So in Fig. 4(b), error bars are not presented. This is different with Fig. 4(a). Take the oscillon (3,3) as an example as well. Since there are three crests, we first obtain the averaged wave height of these three crests. We denote this wave height as A. Then, when four extra experiments under the same physical parameters were done, we can have another four wave heights, denoted by B,C,D,E. Then we average these five wave heights (A–E) and get the averaged wave height, denoted by one square symbol in Fig. 4(a) with the error bar. The almost same wave height but the irregular distribution of the averaged crest-to-crest distance strongly suggest that the observed oscillons should be highly localized so that they are a combination of the elementary oscillons (1,1) and/or (1,2), as shown in Fig. 5. For example, the oscillon (2,2) can be regarded as a combination of the two (1,1) oscillons, i.e., $(2,2) = (1,1) + (1,1)$, which can be close enough since each (1,1) oscillon is highly localized. Similarly, (3,2) can be regarded as a combination of one (1,1) oscillon and one (2,1) oscillon, i.e., $(3,2) = (1,1) + (2,1)$, which are close enough. In addition, (3,3) can be regarded as a combination of the close oscillons (2,1) and (1,2), i.e., $(3,3) = (2,1) + (1,2)$. Since the elementary oscillons (1,1) and (1,2) have the same averaged wave height, as shown in Fig. 4(a), it is easy to explain why the more complicated oscillons like (2,2) and (2,3) have the same wave height. However, since the distance between each (1,1) and/or (1,2) elementary oscillon can be different due to the experimental uncertainty of the initial disturbance on free surface, the so-called averaged crest-to-crest distance should be irregular with large gap and standard deviation.

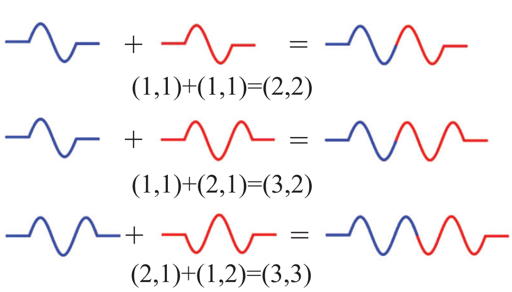
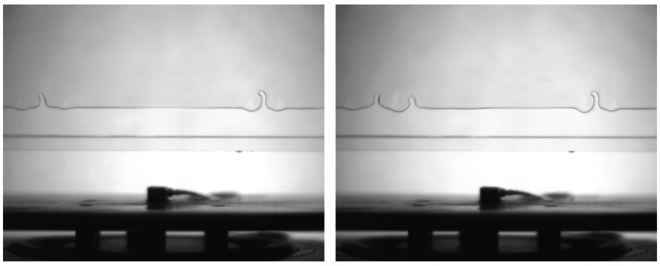


FIG. 5. (Color online) Combinations of the elementary oscillons.



(a) (b)

FIG. 6. High localization of oscillons. Experiments were performed in 15% ethanol-water solution at a frequency of 18 Hz, an acceleration amplitude of 20.503 $\mathrm{m/s^2}$, and a fluid depth of 2 cm. The right oscillon (1,2) remains the same in (a) and (b). The left oscillon is first (1,2) in (a) but then becomes (2,2) in (b) by additional disturbance of the free surface.

Some experiments were performed to support our above viewpoints at a frequency of 18 Hz, an acceleration amplitude of 20.503 m/s^2, and a fluid depth of 2 cm. At first, one stable (1,2) oscillon was excited at the right side of the cell. Then, the flat free surface at the left side of the cell was disturbed to excite another (1,2) oscillon, as shown in Fig. 6(a). It is found that the right (1,2) oscillon remains the same even after the left (1,2) oscillon is excited. Besides, both the left and right (1,2) oscillons have almost the same wave height and shape. After around 1000 oscillation periods, additional disturbance was given to the left (1,2) oscillon, and a new (2,2) oscillon was excited, as shown in Fig. 6(b). It is found that the right (1,2) oscillon remains unchanged in the meanwhile. These highly suggest that the two oscillons on the left and right sides of the cell are independent of each other, say, highly localized. Therefore, our experiments highly suggest that the observed, more complicated (m,n) oscillons reported in this Rapid Communication may be a combination of several (1,1) and/or (1,2) elementary oscillons discovered in [3]. This viewpoint can well explain the experimental results reported above.

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