Nonlinear Dyn (2007) 49:317–318
DOI 10.1007/s11071-006-9136-7

REVIEW ARTICLE

A short communication on Dr. He's modified Lindstedt–Poincaré method

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Received: 12 July 2006 / Accepted: 24 August 2006 / Published online: 29 September 2006

The so-called "modified Lindstedt–Poincaré method" described by Dr. He [1] badly violates the fundamental theorems in calculus and besides contains serious logical paradox.

It is well known that a smooth real function $f(\varepsilon)$ can be expanded into a *unique* Taylor series $f(\varepsilon) = c_0 + c_1\varepsilon + c_2\varepsilon^2 + \cdots c_n\varepsilon^n$, where the coefficient

$$c_k = \frac{1}{k!}\left.\frac{d^k f}{d\varepsilon^k}\right|_{\varepsilon=0}$$

is *independent* of ε (please refer to Fitzpatrick [2], Theorem 41). For example, considering $f(\varepsilon) = \sin\varepsilon$, one has $c_0 = \sin 0 = 0$, $c_1 = \cos 0 = 1$, and so on. In case of $f(\varepsilon) = a$, where a is a constant *independent* of ε, one has certainly $c_0 = a$ and $c_1 = c_2 = c_3 = \cdots = 0$. So, it has no meaning to expand a constant into a Taylor series of ε because all coefficients, except $c_0 = a$, are zero. However, Dr. He expanded a constant into such a Taylor series of ε in [1]. For example, Dr. He expanded the constant zero into the following series

$$0 = \omega^2 + c_1\varepsilon + c_2\varepsilon^2 + \cdots, \tag{A}$$

and Dr. He even obtained a *nonzero* constant $c_1 = -(3/4)A^2$. This badly violates the fundamental theorem in calculus (please refer to Fitzpatrick [2], Theorem 41).

Besides, in the Section 6.17 of the textbook [3], there is such a corollary: *if a power series has a nonzero convergence radius and has a sum that is identically zero, then every coefficient of the series is zero*. Clearly, Dr. He's above expression (A) with nonzero coefficient $c_1 = -(3/4)A^2$ also violates this corollary.

Even if one regards the expression (A) as an asymptotic sequence, all coefficients in (A) must be zero. As pointed out by Nayfeh [4], given an asymptotic sequence $\delta_m(\varepsilon) = \varepsilon^m$, the sequence of $f(\varepsilon)$ in terms of this sequence is *unique*. Besides, using formulae (1.4.16) and (1.4.37) in Nayfeh's book [4], all coefficients of (A) must be zero. Therefore, Dr. He's expression (A) with the nonzero coefficients $c_1 = -(3/4)A^2$ also violates the theorem of perturbation techniques, even if Dr. He regards (A) as an asymptotic sequence.

Therefore, when the constant zero is expand into a series in (A), all coefficients must be zero, no matter either it is a Taylor series or an asymptotic sequence. Thus, it is out of question that Dr. He's expression (A) with the nonzero coefficients $c_1 = -(3/4)A^2$ is completely wrong.

This kind of mistake brings serious logical paradox in Dr. He's approach [1]. Dr. He first rewrote the equation $u'' + \varepsilon u^3 = 0$ into $u'' + 0 \cdot u + \varepsilon u^3 = 0$. Then, substituting (A) and the perturbation expression

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$u = u_0 + \varepsilon u_1 + \varepsilon^2 u_2 + \cdots$ into it, Dr. He has

$$(u_0'' + \omega^2 u_0) + \varepsilon(u_1'' + \omega^2 u_1 + c_1 u_0 + u_0^3) + \cdots.$$

Equating the coefficients of the like power of ε, Dr. He obtained the equations

$$\varepsilon^0: \ u_0'' + \omega^2 u_0 = 0,$$
$$\varepsilon: \ u_1'' + \omega^2 u_1 + c_1 u_0 + u_0^3 = 0,$$

which are exactly Equations (21) and (22) in [1], respectively. It must be pointed out that, to obtain the previous equations, Dr. He *must* assume here that ω^2 is *independent* of ε. Otherwise, he had to expand ω^2 into a power series of ε, and therefore *cannot* obtain the above equations (please refer to Nayfeh [4]). Unfortunately, according to Equation (27) in [1], Dr. He finally obtained such a result $\omega^2 = (3/4)\varepsilon A^2$, which is *dependent* on ε. How can the *same* term ω^2 be first assumed a constant *independent* of ε but finally be given a value *dependent* on ε? So, there exists a serious logical paradox in Dr. He's approach.

Similarly, all of Dr. He's related publications (such as [5, 6]) based on this kind of expansion are completely wrong. This logical paradox comes from the mathematical mistakes that Dr. He expanded a constant into a Taylor series or asymptotic sequence with many *nonzero* coefficients.

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