Supplementary data for "Collisionless periodic orbits in the free-fall three-body problem"

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Table S I. The initial conditions and periods T of the new collisionless periodic free-fall orbits in the case of $\boldsymbol{r}_1(0) = (-0.5, 0)$, $\boldsymbol{r}_2(0) = (0.5, 0)$, $\boldsymbol{r}_3(0) = (x, y)$, $\dot{\boldsymbol{r}}_1(0) = \dot{\boldsymbol{r}}_2(0) = \dot{\boldsymbol{r}}_3(0) = (0, 0)$ and the Newtonian constant of gravitation $G = 1$.

$F_i(m_1, m_2, m_3)$	x	y	T
$F_1(1, 1, 1)$	0.0207067154	0.3133550361	2.1740969264
$F_2(1, 1, 1)$	0.2053886532	0.1952668419	1.6896364928
$F_3(1, 1, 1)$	0.056266428	0.4691503375	4.5419125588
$F_4(1, 1, 1)$	0.1846729355	0.5753740774	5.1586391029
$F_5(1, 1, 1)$	0.0880412663	0.5488924176	4.9647695145
$F_6(1, 1, 1)$	0.314233405	0.5384825297	4.8672002993
$F_7(1, 1, 1)$	0.0741834378	0.5324424488	5.4455591108
$F_8(1, 1, 1)$	0.2871136862	0.5252098584	5.1183291764
$F_9(1, 1, 1)$	0.0623175679	0.4902842102	5.1999756815
$F_{10}(1, 1, 1)$	0.3089693008	0.4236727692	4.8914942162
$F_{11}(1, 1, 1)$	0.4006912162	0.3484015305	4.2747107264
$F_{12}(1, 1, 1)$	0.3895924236	0.3466357955	4.2763269534
$F_{13}(1, 1, 1)$	0.1887216677	0.5103459122	5.9705993269
$F_{14}(1, 1, 1)$	0.1938233757	0.6063751102	7.6482262218
$F_{15}(1, 1, 1)$	0.2574316821	0.363199197	6.4040019625
$F_{16}(1, 1, 1)$	0.1472358808	0.3114646878	6.5387455881
$F_{17}(1, 1, 1)$	0.1403948993	0.5079077887	8.1068148513
$F_{18}(1, 1, 1)$	0.2104156341	0.1080829305	5.3952449725
$F_{19}(1, 1, 1)$	0.0410117513	0.1882022887	8.101047418
$F_{20}(1, 1, 1)$	0.0548189646	0.2383162534	8.8215019091
$F_{21}(1, 1, 1)$	0.2760396991	0.0985020359	5.4225040686
$F_{22}(1, 1, 1)$	0.2662035004	0.0764101615	5.4473716994
$F_{23}(1, 1, 1)$	0.0135939022	0.1210361097	8.4973950159
$F_{24}(1, 1, 1)$	0.0616197629	0.2644961757	9.4244325344
$F_{25}(1, 1, 1)$	0.2927953055	0.1170062116	5.7810404779
$F_{26}(1, 1, 1)$	0.0327160645	0.231329184	10.0217304818
$F_{27}(1, 1, 1)$	0.1523536847	0.1611243738	10.3340878156
$F_{28}(1, 1, 1)$	0.0726030609	0.3037940331	11.8427589787
$F_{29}(1, 1, 1)$	0.3085035769	0.1499949186	7.293882118
$F_{30}(1, 1, 1)$	0.3401651301	0.1168579357	6.1839976588
$F_1(1, 0.8, 0.8)$	0.0009114239	0.3019805958	1.8286248401
$F_2(1, 0.8, 0.8)$	0.0554096945	0.4358787847	2.679626506
$F_3(1, 0.8, 0.8)$	0.314708305	0.3171002382	2.5504611351
$F_4(1, 0.8, 0.8)$	0.0470065136	0.4047349033	3.1695717646
$F_5(1, 0.8, 0.8)$	0.0428491303	0.3894443945	3.6573170641
$F_6(1, 0.8, 0.8)$	0.1080771428	0.5456908479	5.5072190289
$F_7(1, 0.8, 0.8)$	0.1942960822	0.2920607463	3.9686010634
$F_8(1, 0.8, 0.8)$	0.0403354335	0.380070111	4.1385691346
$F_9(1, 0.8, 0.8)$	0.2016807138	0.3445130538	4.4274165052
$F_{10}(1, 0.8, 0.8)$	0.1885308478	0.3499578547	4.9200108461
$F_{11}(1, 0.8, 0.8)$	0.0386388457	0.37364164	4.6139352019
$F_{12}(1, 0.8, 0.8)$	0.2604939376	0.4347691928	5.5847931336
$F_{13}(1, 0.8, 0.8)$	0.1947764043	0.3822990161	5.7130074726
$F_{14}(1, 0.8, 0.8)$	0.1181940386	0.5724230518	8.1600955849
$F_{15}(1, 0.8, 0.8)$	0.3253474955	0.1256079633	3.2110917765
$F_{16}(1, 0.8, 0.8)$	0.4292485918	0.2660046008	4.2565848643
$F_{17}(1, 0.8, 0.8)$	0.0075986006	0.1288965886	5.1331657257
$F_{18}(1, 0.8, 0.8)$	0.3701389869	0.1898186752	3.5781033123
$F_{19}(1, 0.8, 0.8)$	0.1691814162	0.1508888822	5.9395400603
$F_{20}(1, 0.8, 0.8)$	0.2150366646	0.4048515918	8.5571805321

Table S II. The initial conditions and periods T of the new collisionless periodic free-fall orbits in the case of $\boldsymbol{r}_1(0) = (-0.5, 0)$, $\boldsymbol{r}_2(0) = (0.5, 0)$, $\boldsymbol{r}_3(0) = (x, y)$, $\dot{\boldsymbol{r}}_1(0) = \dot{\boldsymbol{r}}_2(0) = \dot{\boldsymbol{r}}_3(0) = (0, 0)$ and the Newtonian constant of gravitation $G = 1$.

$F_i(m_1, m_2, m_3)$	x	y	T
$F_{21}(1, 0.8, 0.8)$	0.247066563	0.0780584462	5.1791534733
$F_{22}(1, 0.8, 0.8)$	0.1505569937	0.1515667886	7.5382540932
$F_{23}(1, 0.8, 0.8)$	0.1674779976	0.1427324464	7.7585512519
$F_{24}(1, 0.8, 0.8)$	0.1527845023	0.068494294	7.5001089956
$F_{25}(1, 0.8, 0.8)$	0.1500221997	0.0630122343	9.2986382763
$F_{26}(1, 0.8, 0.8)$	0.1830123116	0.6092070111	18.8535978723
$F_{27}(1, 0.8, 0.8)$	0.1464997313	0.0473029499	11.0720042333
$F_{28}(1, 0.8, 0.8)$	0.1624065612	0.138489367	13.3138834463
$F_{29}(1, 0.8, 0.8)$	0.1547961371	0.1653004219	17.5487952899
$F_1(1, 0.8, 0.6)$	0.0445314006	0.7402164268	2.8030997992
$F_2(1, 0.8, 0.6)$	0.0596000878	0.7375612563	3.4959622385
$F_3(1, 0.8, 0.6)$	0.1174556037	0.716752919	2.7369656923
$F_4(1, 0.8, 0.6)$	0.0651532615	0.7533349173	4.1668179824
$F_5(1, 0.8, 0.6)$	0.129088109	0.4010761427	2.500764871
$F_6(1, 0.8, 0.6)$	0.3133248745	0.3258575422	2.6081696286
$F_7(1, 0.8, 0.6)$	0.007896547	0.2369793668	2.560892572
$F_8(1, 0.8, 0.6)$	0.0666851373	0.7642377831	4.796190851
$F_9(1, 0.8, 0.6)$	0.1288237799	0.257598905	2.563208106
$F_{10}(1, 0.8, 0.6)$	0.1156599683	0.4136953992	3.1416626327
$F_{11}(1, 0.8, 0.6)$	0.0119593475	0.2408362224	2.9430485048
$F_{12}(1, 0.8, 0.6)$	0.1023506488	0.408866949	3.6750954828
$F_{13}(1, 0.8, 0.6)$	0.169463742	0.0159900098	2.4177885604
$F_{14}(1, 0.8, 0.6)$	0.2707648383	0.1015364734	2.0709517775
$F_{15}(1, 0.8, 0.6)$	0.361264345	0.294301541	2.7889383936
$F_{16}(1, 0.8, 0.6)$	0.4108603284	0.2242851684	2.1055583251
$F_{17}(1, 0.8, 0.6)$	0.0932368548	0.4030127578	4.1731767189
$F_{18}(1, 0.8, 0.6)$	0.1352566765	0.6807175103	6.7293787182
$F_{19}(1, 0.8, 0.6)$	0.2725304223	0.1732922849	2.6175552797
$F_{20}(1, 0.8, 0.6)$	0.3125456686	0.086616029	1.9950219564
$F_{21}(1, 0.8, 0.6)$	0.1882144839	0.3345735824	3.5219440458
$F_{22}(1, 0.8, 0.6)$	0.0716145199	0.0915187956	3.8642156146
$F_{23}(1, 0.8, 0.6)$	0.0869005363	0.397993724	4.6539481356
$F_{24}(1, 0.8, 0.6)$	0.3563440713	0.2981031724	3.6559375422
$F_{25}(1, 0.8, 0.6)$	0.2880532139	0.4825865222	4.6073152572
$F_{26}(1, 0.8, 0.6)$	0.4160772504	0.2310953494	2.8622355529
$F_{27}(1, 0.8, 0.6)$	0.3987945492	0.3025049316	3.3736146266
$F_{28}(1, 0.8, 0.6)$	0.077642587	0.1746563917	4.4628708319
$F_{29}(1, 0.8, 0.6)$	0.0599809897	0.0926220002	4.3653925159
$F_{30}(1, 0.8, 0.6)$	0.0579903544	0.0443882673	4.3050365191
$F_{31}(1, 0.8, 0.6)$	0.09346147	0.0471665194	4.1680025233
$F_{32}(1, 0.8, 0.6)$	0.1325902103	0.4123819959	5.6679076918
$F_{33}(1, 0.8, 0.6)$	0.104144912	0.1482283553	4.3424138584
$F_{34}(1, 0.8, 0.6)$	0.1993569698	0.0777479367	3.8425401785
$F_{35}(1, 0.8, 0.6)$	0.1341720545	0.4004224629	5.4787823377
$F_{36}(1, 0.8, 0.6)$	0.2974062043	0.5279669906	6.3977820308
$F_{37}(1, 0.8, 0.6)$	0.2979333584	0.4813611322	5.0215788607
$F_{38}(1, 0.8, 0.6)$	0.2660209853	0.507899546	6.132677999
$F_{39}(1, 0.8, 0.6)$	0.352368316	0.2982025789	4.0622574955
$F_{40}(1, 0.8, 0.6)$	0.4161870623	0.232018837	3.1969547246

Table S III. The initial conditions and periods T of the new collisionless periodic free-fall orbits in the case of $\boldsymbol{r}_1(0) = (-0.5, 0)$, $\boldsymbol{r}_2(0) = (0.5, 0)$, $\boldsymbol{r}_3(0) = (x, y)$, $\dot{\boldsymbol{r}}_1(0) = \dot{\boldsymbol{r}}_2(0) = \dot{\boldsymbol{r}}_3(0) = (0, 0)$ and the Newtonian constant of gravitation $G = 1$.

$F_i(m_1, m_2, m_3)$	x	y	T
$F_{41}(1, 0.8, 0.6)$	0.4020135877	0.3030079422	3.6862253065
$F_{42}(1, 0.8, 0.6)$	0.1226012128	0.3700294495	5.7506652807
$F_{43}(1, 0.8, 0.6)$	0.1005984521	0.1891151438	4.8651314925
$F_{44}(1, 0.8, 0.6)$	0.2712808805	0.1853534625	4.0744291974
$F_{45}(1, 0.8, 0.6)$	0.1681172791	0.3751181431	5.746011321
$F_{46}(1, 0.8, 0.6)$	0.2156072166	0.0352712952	3.9293028391
$F_{47}(1, 0.8, 0.6)$	0.2256928469	0.0765660306	3.8667808099
$F_{48}(1, 0.8, 0.6)$	0.1370635146	0.4370979031	6.4419369834
$F_{49}(1, 0.8, 0.6)$	0.2751838416	0.5445094738	7.2105813537
$F_{50}(1, 0.8, 0.6)$	0.1171739128	0.2424690464	5.0831185087
$F_{51}(1, 0.8, 0.6)$	0.1845105186	0.4501096494	6.2787672531
$F_{52}(1, 0.8, 0.6)$	0.4158959655	0.2325668494	3.5206991506
$F_{53}(1, 0.8, 0.6)$	0.1631890832	0.2103797105	5.2589880358
$F_{54}(1, 0.8, 0.6)$	0.1983940001	0.4025046079	6.3528167889
$F_{55}(1, 0.8, 0.6)$	0.2278308853	0.5414697914	7.8346766349
$F_{56}(1, 0.8, 0.6)$	0.1400273104	0.0081786581	4.6831272984
$F_{57}(1, 0.8, 0.6)$	0.2396969281	0.6174793436	8.4831505812
$F_{58}(1, 0.8, 0.6)$	0.3162910832	0.1332526561	3.5796429232
$F_{59}(1, 0.8, 0.6)$	0.2092641475	0.4289510221	6.992742159
$F_{60}(1, 0.8, 0.6)$	0.2451298945	0.5563343875	8.6490881257
$F_{61}(1, 0.8, 0.6)$	0.2033682605	0.5895584432	8.8821315833
$F_{62}(1, 0.8, 0.6)$	0.0792440005	0.0978998982	6.2066610525
$F_{63}(1, 0.8, 0.6)$	0.1798006367	0.5682045433	9.3055595206
$F_{64}(1, 0.8, 0.6)$	0.1635781743	0.5519695391	9.7486824006
$F_{65}(1, 0.8, 0.6)$	0.1496991995	0.1853063639	7.1021694436
$F_{66}(1, 0.8, 0.6)$	0.1517569566	0.5393156268	10.2043588748
$F_{67}(1, 0.8, 0.6)$	0.2762432619	0.1953715885	6.2775182238
$F_{68}(1, 0.8, 0.6)$	0.1707850755	0.0856593151	6.9891771436
$F_{69}(1, 0.8, 0.6)$	0.1843011931	0.2329050114	10.3627717531
$F_1(1, 0.8, 0.4)$	0.0907814512	0.4017862494	2.6296980224
$F_2(1, 0.8, 0.4)$	0.0731681909	0.6154884228	3.9909158705
$F_3(1, 0.8, 0.4)$	0.1643021746	0.3586184694	2.8687541956
$F_4(1, 0.8, 0.4)$	0.0820569602	0.4346030503	3.2699876372
$F_5(1, 0.8, 0.4)$	0.3039731933	0.2570037825	2.4050767593
$F_6(1, 0.8, 0.4)$	0.003823003	0.5336567474	3.7553714566
$F_7(1, 0.8, 0.4)$	0.2215676344	0.3878099512	3.3450631915
$F_8(1, 0.8, 0.4)$	0.3334492443	0.2914625638	2.9491668861
$F_9(1, 0.8, 0.4)$	0.0734875559	0.430759221	3.7612586546
$F_{10}(1, 0.8, 0.4)$	0.0041432429	0.3565873877	3.3430163241
$F_{11}(1, 0.8, 0.4)$	0.364734727	0.2936105654	2.8392914831
$F_{12}(1, 0.8, 0.4)$	0.3400677332	0.3044773669	3.4135118241
$F_{13}(1, 0.8, 0.4)$	0.065470114	0.3237298022	4.9143582571
$F_{14}(1, 0.8, 0.4)$	0.0683822051	0.4258256814	4.2186879217
$F_{15}(1, 0.8, 0.4)$	0.3051209633	0.1717154384	2.6362825973
$F_{16}(1, 0.8, 0.4)$	0.381819046	0.2165579773	2.6549491894
$F_{17}(1, 0.8, 0.4)$	0.3406869343	0.2557425196	2.9271614571
$F_{18}(1, 0.8, 0.4)$	0.3519249675	0.1805278285	2.3962764398
$F_{19}(1, 0.8, 0.4)$	0.0067573659	0.3551234191	3.7006572503
$F_{20}(1, 0.8, 0.4)$	0.3966555518	0.2200475447	2.5793846625

Table S IV. The initial conditions and periods T of the new collisionless periodic free-fall orbits in the case of $\boldsymbol{r}_1(0) = (-0.5, 0)$, $\boldsymbol{r}_2(0) = (0.5, 0)$, $\boldsymbol{r}_3(0) = (x, y)$, $\dot{\boldsymbol{r}}_1(0) = \dot{\boldsymbol{r}}_2(0) = \dot{\boldsymbol{r}}_3(0) = (0, 0)$ and the Newtonian constant of gravitation $G = 1$.

$F_i(m_1, m_2, m_3)$	x	y	T
$F_{21}(1, 0.8, 0.4)$	0.3377184031	0.3084466213	3.8289301499
$F_{22}(1, 0.8, 0.4)$	0.2559808544	0.3985035719	4.1088019361
$F_{23}(1, 0.8, 0.4)$	0.3919889126	0.2253938155	3.0196263432
$F_{24}(1, 0.8, 0.4)$	0.0650272018	0.4217643778	4.6597662391
$F_{25}(1, 0.8, 0.4)$	0.3507035957	0.1514721792	2.5093279573
$F_{26}(1, 0.8, 0.4)$	0.0083551748	0.3537463108	4.0436510812
$F_{27}(1, 0.8, 0.4)$	0.083924021	0.3307729197	5.3174336486
$F_{28}(1, 0.8, 0.4)$	0.1534813802	0.3445489508	5.3109746054
$F_{29}(1, 0.8, 0.4)$	0.2639061633	0.4003107771	4.4615936588
$F_{30}(1, 0.8, 0.4)$	0.3968538929	0.2304351014	3.36131345
$F_{31}(1, 0.8, 0.4)$	0.3332373486	0.3087120049	4.2145144195
$F_{32}(1, 0.8, 0.4)$	0.4200988856	0.2278457172	3.100244422
$F_{33}(1, 0.8, 0.4)$	0.4130847007	0.1836206658	2.7777999782
$F_{34}(1, 0.8, 0.4)$	0.0626424176	0.4185140509	5.0903858286
$F_{35}(1, 0.8, 0.4)$	0.2968561297	0.1524320976	3.0849296562
$F_{36}(1, 0.8, 0.4)$	0.1204686367	0.3718569619	5.9520762241
$F_{37}(1, 0.8, 0.4)$	0.3388963416	0.4809266124	6.4469817763
$F_{38}(1, 0.8, 0.4)$	0.3987797585	0.2334398833	3.6879026872
$F_{39}(1, 0.8, 0.4)$	0.3289050765	0.3077086141	4.5828783734
$F_{40}(1, 0.8, 0.4)$	0.4211234979	0.1897019745	3.3348881047
$F_{41}(1, 0.8, 0.4)$	0.1740093467	0.4061799018	6.945954866
$F_{42}(1, 0.8, 0.4)$	0.4229990097	0.1914129869	3.6005720033
$F_{43}(1, 0.8, 0.4)$	0.1491465141	0.1587305525	5.9110461898
$F_{44}(1, 0.8, 0.4)$	0.1071012257	0.1862114119	7.0200258193
$F_1(1, 0.8, 0.2)$	0.0247367455	0.5956711592	2.9388503785
$F_2(1, 0.8, 0.2)$	0.0788881323	0.7268131374	3.0188278709
$F_3(1, 0.8, 0.2)$	0.0213288594	0.6061184155	3.434230462
$F_4(1, 0.8, 0.2)$	0.0726486463	0.3079938956	2.7012372544
$F_5(1, 0.8, 0.2)$	0.0191076293	0.6201117694	3.8749764277
$F_6(1, 0.8, 0.2)$	0.1310631652	0.3036588095	2.9464698551
$F_7(1, 0.8, 0.2)$	0.0071275432	0.4202963121	3.229371033
$F_8(1, 0.8, 0.2)$	0.0578321783	0.4378406225	3.3345102032
$F_9(1, 0.8, 0.2)$	0.32805122	0.4258152565	4.5412446489
$F_{10}(1, 0.8, 0.2)$	0.0107850066	0.412951734	3.515266184
$F_{11}(1, 0.8, 0.2)$	0.0518431819	0.4240660566	3.6691833705
$F_{12}(1, 0.8, 0.2)$	0.2677953092	0.1910152983	2.7646135824
$F_{13}(1, 0.8, 0.2)$	0.3185180627	0.2611918166	3.017887689
$F_{14}(1, 0.8, 0.2)$	0.0161474528	0.6441659117	4.6837208272
$F_{15}(1, 0.8, 0.2)$	0.0957979866	0.7922510102	4.8571260165
$F_{16}(1, 0.8, 0.2)$	0.2973457769	0.2160446151	3.0341335482
$F_{17}(1, 0.8, 0.2)$	0.3192685287	0.1435620243	2.6213636288
$F_{18}(1, 0.8, 0.2)$	0.0151043114	0.6535617498	5.0657847438
$F_{19}(1, 0.8, 0.2)$	0.2809348051	0.4215890462	5.6828932918
$F_{20}(1, 0.8, 0.2)$	0.3505735652	0.1838631164	2.7916720023
$F_{21}(1, 0.8, 0.2)$	0.0835830692	0.3734731952	5.6217700487
$F_{22}(1, 0.8, 0.2)$	0.227120654	0.3551247364	3.8574172516
$F_{23}(1, 0.8, 0.2)$	0.3402615147	0.1637450999	2.8506619375
$F_{24}(1, 0.8, 0.2)$	0.3793454984	0.2079457559	3.102833336
$F_{25}(1, 0.8, 0.2)$	0.3188392408	0.2307237833	3.4919940375

Table S V. The initial conditions and periods T of the new collisionless periodic free-fall orbits in the case of $\boldsymbol{r}_1(0) = (-0.5, 0)$, $\boldsymbol{r}_2(0) = (0.5, 0)$, $\boldsymbol{r}_3(0) = (x, y)$, $\dot{\boldsymbol{r}}_1(0) = \dot{\boldsymbol{r}}_2(0) = \dot{\boldsymbol{r}}_3(0) = (0, 0)$ and the Newtonian constant of gravitation $G = 1$.

$F_i(m_1, m_2, m_3)$	x	y	T
$F_{26}(1, 0.8, 0.2)$	0.2365256725	0.3585230897	4.1103828594
$F_{27}(1, 0.8, 0.2)$	0.4062681391	0.1740808481	3.1511604419
$F_{28}(1, 0.8, 0.2)$	0.2121623131	0.6190535787	9.7560815483
$F_{29}(1, 0.8, 0.2)$	0.4109560735	0.1434385455	2.9753651036
$F_{30}(1, 0.8, 0.2)$	0.2875973419	0.5233120555	10.9328732255
$F_{31}(1, 0.8, 0.2)$	0.2145732729	0.578482645	9.5431888036
$F_{32}(1, 0.8, 0.2)$	0.4217142944	0.1509680324	3.1849044537
$F_1(0.6, 0.8, 1)$	0.2061730906	0.4463433325	2.6925803371
$F_2(0.6, 0.8, 1)$	0.2632585995	0.4021064128	2.8891085282
$F_3(0.6, 0.8, 1)$	0.3087699287	0.281712961	2.2526299455
$F_4(0.6, 0.8, 1)$	0.3064570616	0.2486099306	2.3813615017
$F_5(0.6, 0.8, 1)$	0.2810833582	0.4016924931	3.2814736858
$F_6(0.6, 0.8, 1)$	0.3226484475	0.181377391	1.9623770921
$F_7(0.6, 0.8, 1)$	0.2687327027	0.0903053982	1.9434750648
$F_8(0.6, 0.8, 1)$	0.323395942	0.1697487157	2.1207316616
$F_9(0.6, 0.8, 1)$	0.3271831808	0.2547968571	2.62803189
$F_{10}(0.6, 0.8, 1)$	0.289925259	0.4030770616	3.6787912077
$F_{11}(0.6, 0.8, 1)$	0.342626207	0.1830843562	2.3182580914
$F_{12}(0.6, 0.8, 1)$	0.3372214876	0.260134566	2.9032413179
$F_{13}(0.6, 0.8, 1)$	0.333942851	0.1166303462	1.9513533175
$F_{14}(0.6, 0.8, 1)$	0.3514438378	0.1374554522	2.133577047
$F_{15}(0.6, 0.8, 1)$	0.3541926333	0.1912396242	2.5272612515
$F_{16}(0.6, 0.8, 1)$	0.2137677527	0.422342585	5.8220417242
$F_{17}(0.6, 0.8, 1)$	0.3635347723	0.1483383694	2.3025570478
$F_{18}(0.6, 0.8, 1)$	0.0697898821	0.3190727633	6.2973444729
$F_{19}(0.6, 0.8, 1)$	0.3326592654	0.2979880796	4.4551677666
$F_{20}(0.6, 0.8, 1)$	0.0774214943	0.2835443503	5.903006977
$F_{21}(0.6, 0.8, 1)$	0.0779210325	0.3163212877	6.1187532147
$F_{22}(0.6, 0.8, 1)$	0.1428136093	0.191027388	5.3746485278
$F_{23}(0.6, 0.8, 1)$	0.0894008417	0.2215222207	6.0165906664
$F_{24}(0.6, 0.8, 1)$	0.041186859	0.138693905	5.7982241905
$F_{25}(0.6, 0.8, 1)$	0.2645032395	0.3738186198	6.4584844333
$F_{26}(0.6, 0.8, 1)$	0.0876902685	0.1043044639	5.9308332463
$F_{27}(0.6, 0.8, 1)$	0.1201933414	0.4542911576	9.4434059349
$F_{28}(0.6, 0.8, 1)$	0.3461730153	0.201836099	4.1324912323
$F_{29}(0.6, 0.8, 1)$	0.1715954664	0.2463490645	8.1331787201
$F_{30}(0.6, 0.8, 1)$	0.1446319096	0.4773197126	12.5809129586

Table S VI. The initial conditions and periods T of the new collisionless periodic free-fall orbits in the case of $\boldsymbol{r}_1(0) = (-0.5, 0)$, $\boldsymbol{r}_2(0) = (0.5, 0)$, $\boldsymbol{r}_3(0) = (x, y)$, $\dot{\boldsymbol{r}}_1(0) = \dot{\boldsymbol{r}}_2(0) = \dot{\boldsymbol{r}}_3(0) = (0, 0)$ and the Newtonian constant of gravitation $G = 1$.

$F_i(m_1, m_2, m_3)$	x	y	T
$F_1(1, 0.412, 0.267)$	0.0696185837	0.2658051873	2.6985404348
$F_2(1, 0.412, 0.267)$	0.2370426572	0.1425757376	2.7241294479
$F_3(1, 0.412, 0.267)$	0.0146667391	0.1839445224	3.1237052493
$F_4(1, 0.412, 0.267)$	0.0833137102	0.3201850026	3.1738762925
$F_5(1, 0.412, 0.267)$	0.1694423202	0.2129138499	2.7611666617
$F_6(1, 0.412, 0.267)$	0.3090242251	0.1286410243	2.7026918156
$F_7(1, 0.412, 0.267)$	0.3497142937	0.3970315747	3.794104897
$F_8(1, 0.412, 0.267)$	0.3562226518	0.1251087062	2.6556242265
$F_9(1, 0.412, 0.267)$	0.3352635523	0.0781567667	2.7807277863
$F_{10}(1, 0.412, 0.267)$	0.3845688061	0.116062387	2.5901805561
$F_{11}(1, 0.412, 0.267)$	0.3622341209	0.3946501198	4.1233946066
$F_{12}(1, 0.412, 0.267)$	0.0446536477	0.4294152856	7.1578680967
$F_{13}(1, 0.412, 0.267)$	0.4029604686	0.1069664133	2.5269918268
$F_{14}(1, 0.412, 0.267)$	0.0100620946	0.0512147802	5.9173163705
$F_{15}(1, 0.412, 0.267)$	0.125667006	0.239131378	9.3079842374
$F_1(0.393, 0.288, 0.662)$	0.1640136783	0.6735401	7.812437591
$F_2(0.393, 0.288, 0.662)$	0.1261267815	0.3269672542	5.2445930336
$F_3(0.393, 0.288, 0.662)$	0.0916017197	0.3035508004	5.8282287397
$F_4(0.393, 0.288, 0.662)$	0.1486932205	0.2100396153	5.3029350919
$F_5(0.393, 0.288, 0.662)$	0.1187231428	0.4012184719	6.2437180885
$F_6(0.393, 0.288, 0.662)$	0.1182420436	0.1846790135	5.092728117
$F_7(0.393, 0.288, 0.662)$	0.2887660709	0.2450525488	4.6269133219
$F_8(0.393, 0.288, 0.662)$	0.1019954706	0.2381036852	5.5416092565
$F_9(0.393, 0.288, 0.662)$	0.0766816043	0.3069970559	6.5082868583
$F_{10}(0.393, 0.288, 0.662)$	0.2606714907	0.2102234038	5.0203486601
$F_{11}(0.393, 0.288, 0.662)$	0.0757893679	0.2133021062	6.0590569238
$F_{12}(0.393, 0.288, 0.662)$	0.1116893838	0.3554295349	6.5128017143
$F_{13}(0.393, 0.288, 0.662)$	0.1125608361	0.1224363089	5.3628247444
$F_{14}(0.393, 0.288, 0.662)$	0.1178075355	0.0661495433	5.5132385113
$F_{15}(0.393, 0.288, 0.662)$	0.0687502433	0.3118270626	7.1658475962
$F_{16}(0.393, 0.288, 0.662)$	0.0647208635	0.2139071765	6.6521212276
$F_{17}(0.393, 0.288, 0.662)$	0.069510492	0.1641819867	6.5076632342
$F_{18}(0.393, 0.288, 0.662)$	0.230959903	0.5765600406	11.5959431914
$F_{19}(0.393, 0.288, 0.662)$	0.2128942728	0.4947916798	16.1336767755

Table S VII. The initial conditions and periods T of the new collisionless periodic free-fall orbits in the case of $r_1(0) = (-0.5, 0)$, $r_2(0) = (0.5, 0)$, $r_3(0) = (x, y)$, $\dot{r}_1(0) = \dot{r}_2(0) = \dot{r}_3(0) = (0, 0)$ and the Newtonian constant of gravitation $G = 1$.

$F_i(m_1, m_2, m_3)$	x	y	T
$F_1(1, 0.1, 0.01)$	0.2637098062	0.3223452642	4.1240304796
$F_2(1, 0.1, 0.01)$	0.4048310725	0.2237038779	4.1716470072
$F_3(1, 0.1, 0.01)$	0.2147782008	0.1077093168	3.9790466582
$F_4(1, 0.1, 0.01)$	0.2078032484	0.2264870718	7.5313923445
$F_5(1, 0.1, 0.01)$	0.3420646652	0.1816680909	4.1745406609
$F_6(1, 0.1, 0.01)$	0.322285491	0.3898268332	4.2372139628
$F_7(1, 0.1, 0.01)$	0.3098125586	0.455675641	4.7504575927
$F_8(1, 0.1, 0.01)$	0.4437913567	0.2599775981	4.2722696008
$F_9(1, 0.1, 0.01)$	0.3740521913	0.091996364	4.1164154808
$F_{10}(1, 0.1, 0.01)$	0.4012436381	0.2297770587	4.2762386311
$F_{11}(1, 0.1, 0.01)$	0.2040726991	0.434120237	8.1904427859
$F_{12}(1, 0.1, 0.01)$	0.3939175296	0.0553626843	4.0683775952
$F_{13}(1, 0.1, 0.01)$	0.3798322667	0.2197844843	4.3022945648
$F_{14}(1, 0.1, 0.01)$	0.3855901813	0.1127238841	4.1808409644
$F_{15}(1, 0.1, 0.01)$	0.1232124435	0.5137380415	11.8303957904
$F_{16}(1, 0.1, 0.01)$	0.3971918911	0.0698418327	4.1319789315
$F_{17}(1, 0.1, 0.01)$	0.4343256819	0.2548614762	4.3508742335
$F_{18}(1, 0.1, 0.01)$	0.3976903382	0.1273093742	4.2361519002
$F_{19}(1, 0.1, 0.01)$	0.4792863654	0.1722845911	4.2684810269
$F_{20}(1, 0.1, 0.01)$	0.4063632838	0.2351046124	4.3625580824
$F_{21}(1, 0.1, 0.01)$	0.0286367943	0.3812816435	8.5047496277
$F_{22}(1, 0.1, 0.01)$	0.4332394364	0.3299600942	8.4598752219
$F_{23}(1, 0.1, 0.01)$	0.4076791773	0.0467476223	4.0919100798
$F_{24}(1, 0.1, 0.01)$	0.4049930302	0.0833438752	4.1846220342
$F_{25}(1, 0.1, 0.01)$	0.4073178185	0.1365051004	4.2841096804
$F_{26}(1, 0.1, 0.01)$	0.3973188682	0.2318063189	4.3927459955
$F_{27}(1, 0.1, 0.01)$	0.0245798454	0.1124115436	7.8652084557
$F_{28}(1, 0.1, 0.01)$	0.1791861925	0.5509874774	12.5084848664
$F_{29}(1, 0.1, 0.01)$	0.1628157434	0.2202038256	14.8951228097
$F_{30}(1, 0.1, 0.01)$	0.1806588084	0.2818709164	11.9471473781
$F_{31}(1, 0.1, 0.01)$	0.1976836514	0.0796205239	8.4904949178
$F_{32}(1, 0.1, 0.01)$	0.1639685003	0.1090252721	8.2476103811
$F_{33}(1, 0.1, 0.01)$	0.0894432713	0.1239594275	8.3297195279
$F_{34}(1, 0.1, 0.01)$	0.4498032667	0.1955132276	8.7976059008
$F_{35}(1, 0.1, 0.01)$	0.4184093885	0.0405120833	4.1052167183
$F_{36}(1, 0.1, 0.01)$	0.4172447254	0.0667343178	4.1876070995
$F_{37}(1, 0.1, 0.01)$	0.4195574344	0.1004418013	4.2739491568
$F_{38}(1, 0.1, 0.01)$	0.1609403765	0.2133699551	18.3229815686
$F_{39}(1, 0.1, 0.01)$	0.115184335	0.4977120676	19.4685172563
$F_{40}(1, 0.1, 0.01)$	0.0443237976	0.6234684348	16.4555994451
$F_{41}(1, 0.1, 0.01)$	0.102986919	0.6408407394	13.6719634393
$F_{42}(1, 0.1, 0.01)$	0.2523008228	0.3644016559	16.042631182
$F_{43}(1, 0.1, 0.01)$	0.1209203308	0.4812754993	22.9962195266
$F_{44}(1, 0.1, 0.01)$	0.2586962714	0.3812472776	23.1901982805
$F_{45}(1, 0.1, 0.01)$	0.0779041585	0.5405441704	24.4050051789
$F_{46}(1, 0.1, 0.01)$	0.0934633801	0.1602180447	23.021605836
$F_{47}(1, 0.1, 0.01)$	0.0037334911	0.5374456436	24.1357073928
$F_{48}(1, 0.1, 0.01)$	0.0956835072	0.4816819311	31.0259570558

Table S VIII. The free group elements for the periodic three-body orbits

$F_i(m_1, m_2, m_3)$	free group element
$F_1(1,1,1)$	AbABbaBa
$F_2(1,1,1)$	ABaBbAba
$F_3(1,1,1)$	AbAbABaAbaBaBa
$F_4(1,1,1)$	AbABBaaAAbbaBa
$F_5(1,1,1)$	AbabABaAbaBABa
$F_6(1,1,1)$	AbaBAbaABabABa
$F_7(1,1,1)$	AbaaBAbbBBabAABa
$F_8(1,1,1)$	AbaBBabAaBAbbABa
$F_9(1,1,1)$	AbAbABaaAAbaBaBa
$F_{10}(1,1,1)$	AbaBBBabAaBAbbbABa
$F_{11}(1,1,1)$	AAbABBAbaABabbaBaa
$F_{12}(1,1,1)$	AAbABBBaaAAbbbaBaa
$F_{13}(1,1,1)$	AbABBBaaBaAbAAbbbaBa
$F_{14}(1,1,1)$	AbABAbaBabBAbABabaBa
$F_{15}(1,1,1)$	AbaBBAbaBaBAbBabAbABabbABa
$F_{16}(1,1,1)$	AbaBBABaaaBAbBabAAAbabbABa
$F_{17}(1,1,1)$	AbABBAbaBaBAbBabAbABabbaBa
$F_{18}(1,1,1)$	AAbABBBBBBBBAbAAaaBabbbbbbbbbaBaa
$F_{19}(1,1,1)$	AbABBBBBAbaaaaabAbBaBAAAAABabbbbbaBa
$F_{20}(1,1,1)$	AbABBBBBAbaaaaaabAbBaBAAAAAABabbbbbaBa
$F_{21}(1,1,1)$	AAAbABBBBBBBBBBAbAAaaBabbbbbbbbbbbaBaaa
$F_{22}(1,1,1)$	AAAbABBBBBBBBBBAbAAAaaaBabbbbbbbbbbbaBaaa
$F_{23}(1,1,1)$	AbABBBBBBBAbaaaaabAbBaBAAAAAABabbbbbbbaBa
$F_{24}(1,1,1)$	AbABBBBBBAbaaaaaaabAbBaBAAAAAAAABabbbbbbaBa
$F_{25}(1,1,1)$	AAAbABBBBBBBBBBBBAbAAAaaaBabbbbbbbbbbbbbaBaaa
$F_{26}(1,1,1)$	AbABBBBBBBAbaaaaaaabAbbBBaBAAAAAAAABabbbbbbbaBa
$F_{27}(1,1,1)$	AbaBABABAbabABABaBabaBABAbaBbABababABABabaBABababABa
$F_{28}(1,1,1)$	AbABBBBBBAbaaaaaaaaaaabAbbBBaBAAAAAAAAAAAABabbbbbaBa
$F_{29}(1,1,1)$	AAAbABBBBBBBBBBBBBBBBBBAbAAAaaaBabbbbbbbbbbbbbbbbbbbaBaaa
$F_{30}(1,1,1)$	AAAAbABBBBBBBBBBBBBBBBBBAbAAAAaaaaBabbbbbbbbbbbbbbbbbbbaBaaaa
$F_1(1,0.8,0.8)$	BaBbAb
$F_2(1,0.8,0.8)$	AbaBbABa
$F_3(1,0.8,0.8)$	AAbABbaBaa
$F_4(1,0.8,0.8)$	AbaBBbbABa
$F_5(1,0.8,0.8)$	AbABBBbbbaBa
$F_6(1,0.8,0.8)$	AbaBAbaABabABa
$F_7(1,0.8,0.8)$	AbaBAbbBBabABa
$F_8(1,0.8,0.8)$	AbABBBBbbbbaBa
$F_9(1,0.8,0.8)$	AbaBBabAaBAbbABa
$F_{10}(1,0.8,0.8)$	AbaBBAbaABabbABa
$F_{11}(1,0.8,0.8)$	AbABBBBBbbbbbaBa
$F_{12}(1,0.8,0.8)$	AbaBBBabAaBAbbbABa
$F_{13}(1,0.8,0.8)$	AbaBBBAbaABabbbABa
$F_{14}(1,0.8,0.8)$	AbaBAbaBBaAbbABabABa
$F_{15}(1,0.8,0.8)$	AAAbaBBAbaABabbABaaa
$F_{16}(1,0.8,0.8)$	AAAbABBAbaABabbaBaaa
$F_{17}(1,0.8,0.8)$	BaBAAAABaBBbbAbaaaabAb
$F_{18}(1,0.8,0.8)$	AAAbaBBBabAaBAbbbABaaa
$F_{19}(1,0.8,0.8)$	AbaBABababABaAbaBABAbabABa
$F_{20}(1,0.8,0.8)$	AbaBBBAbbAbABaAbaBaBBabbbABa

Table S IX. The free group elements for the periodic three-body orbits

$F_i(m_1, m_2, m_3)$	free group element
$F_{21}(1, 0.8, 0.8)$	AABaBBAbabaBAbBabABABabbABaa
$F_{22}(1, 0.8, 0.8)$	AbaBABababABBabAaBABbaBABAbabABa
$F_{23}(1, 0.8, 0.8)$	AbaBABababABABabAaBAbabaBABAbabABa
$F_{24}(1, 0.8, 0.8)$	AbaBABAbabABABBabAaBABabaBABababABa
$F_{25}(1, 0.8, 0.8)$	AbaBABAbabABABababABaAbaBABAbabaBABababABa
$F_{26}(1, 0.8, 0.8)$	AbaBABAbbABaBBabAbaBBaBAbaBBbABabAbbABaBABbABBBbabABa
$F_{27}(1, 0.8, 0.8)$	AbaBABAbabaBABababABABabAaBABababaBABAbabABABBababABa
$F_{28}(1, 0.8, 0.8)$	AbaBABBababABABaBABABabaBABabAaBABabaBABAbabABabaBABABAbabABa
$F_{29}(1, 0.8, 0.8)$	AbaBABababAbabaBABababABAbaBABAbabABaAbaBABababABabaBABAbabABABaBABAbabABa
$F_1(1, 0.8, 0.6)$	AbBa
$F_2(1, 0.8, 0.6)$	AbaABa
$F_3(1, 0.8, 0.6)$	AbAaBa
$F_4(1, 0.8, 0.6)$	AbaaAABa
$F_5(1, 0.8, 0.6)$	AbAbBaBa
$F_6(1, 0.8, 0.6)$	AAbaABaa
$F_7(1, 0.8, 0.6)$	BaBaaAAbAb
$F_8(1, 0.8, 0.6)$	AbaaaAAABa
$F_9(1, 0.8, 0.6)$	AbAbaABaBa
$F_{10}(1, 0.8, 0.6)$	AbAbbBBaBa
$F_{11}(1, 0.8, 0.6)$	BaBaaaAAAbAb
$F_{12}(1, 0.8, 0.6)$	AbAbbbBBBaBa
$F_{13}(1, 0.8, 0.6)$	ABaBaaAAbAba
$F_{14}(1, 0.8, 0.6)$	AABaBaAbAbaa
$F_{15}(1, 0.8, 0.6)$	AAbAbbBBaBaa
$F_{16}(1, 0.8, 0.6)$	AAAbAbBaBaaaA
$F_{17}(1, 0.8, 0.6)$	AbAbbbbBBBBaBa
$F_{18}(1, 0.8, 0.6)$	AbABAbbBBababaBa
$F_{19}(1, 0.8, 0.6)$	AAbAbaaAABaBaa
$F_{20}(1, 0.8, 0.6)$	AAABaBaAbAbaaa
$F_{21}(1, 0.8, 0.6)$	AbABaaaAAAbaBa
$F_{22}(1, 0.8, 0.6)$	AbAbAbAbBaBaBaBa
$F_{23}(1, 0.8, 0.6)$	AbAbbbbbBBBBBaBa
$F_{24}(1, 0.8, 0.6)$	AAbAbbbbBBBBaBaa
$F_{25}(1, 0.8, 0.6)$	AbAbaaaaAAAABaBa
$F_{26}(1, 0.8, 0.6)$	AAAbAbbbBBBaBaaa
$F_{27}(1, 0.8, 0.6)$	AAbAbaaaAAABaBaaA
$F_{28}(1, 0.8, 0.6)$	AbAbabABaAbaBABaBa
$F_{29}(1, 0.8, 0.6)$	AbAbbaBAbBabABBaBa
$F_{30}(1, 0.8, 0.6)$	AbAbbabAbBaBABBaBa
$F_{31}(1, 0.8, 0.6)$	AbaBABABAbAaBababABa
$F_{32}(1, 0.8, 0.6)$	AbAbBaBABbBabAbBaBa
$F_{33}(1, 0.8, 0.6)$	AbaBABabAaBAbabABa
$F_{34}(1, 0.8, 0.6)$	AAbAbAbaBbABaBaBaa
$F_{35}(1, 0.8, 0.6)$	AbAbAbABaAbaBaBaBa
$F_{36}(1, 0.8, 0.6)$	AbABBAbAbBaBabbaBa
$F_{37}(1, 0.8, 0.6)$	AbAbaaaaaAAAAABaBa
$F_{38}(1, 0.8, 0.6)$	AbABABaBaAbAbababaBa
$F_{39}(1, 0.8, 0.6)$	AAbAbbbbbBBBBBaBaa
$F_{40}(1, 0.8, 0.6)$	AAAbAbbbbBBBBaBaaaA
$F_{41}(1, 0.8, 0.6)$	AAbAbaaaaAAAABaBaaA
$F_{42}(1, 0.8, 0.6)$	AbAbabABaaAAbaBABaBa
$F_{43}(1, 0.8, 0.6)$	AbABABaBaaAAbAbababaBa
$F_{44}(1, 0.8, 0.6)$	AAbAbabAbAaBaBABaBaa
$F_{45}(1, 0.8, 0.6)$	AbAbaabAbAaBaBAABaBa

Table S X. The free group elements for the periodic three-body orbits

$F_i(m_1, m_2, m_3)$	free group element
$F_{46}(1, 0.8, 0.6)$	AAbAbbaBAbBabABBaBaa
$F_{47}(1, 0.8, 0.6)$	AAbAbabABaAbaBABaBaa
$F_{48}(1, 0.8, 0.6)$	AbAbBBaBAbBabAbbBaBa
$F_{49}(1, 0.8, 0.6)$	AbABBBAbAbBaBabbbaBa
$F_{50}(1, 0.8, 0.6)$	AbAbaabABaAbaBAABaBa
$F_{51}(1, 0.8, 0.6)$	AbAbaAbAbaABaBaABaBa
$F_{52}(1, 0.8, 0.6)$	AAAbAbbbbbBBBBBaBaaaA
$F_{53}(1, 0.8, 0.6)$	AbaBabaBabAaBAbABABbABa
$F_{54}(1, 0.8, 0.6)$	AbAbaaabAbAaBaBAAABaBa
$F_{55}(1, 0.8, 0.6)$	AbABBBBAbAbBaBabbbbaBa
$F_{56}(1, 0.8, 0.6)$	AbaBAABaBaaAAbAbaabABa
$F_{57}(1, 0.8, 0.6)$	AbABBBBBAbAaBabbbbbaBa
$F_{58}(1, 0.8, 0.6)$	AAAbAbabAbAaBaBABaBaaa
$F_{59}(1, 0.8, 0.6)$	AbAbaaaabAbAaBaBAAAABaBa
$F_{60}(1, 0.8, 0.6)$	AbABBBBAbAbbBBaBabbbbaBa
$F_{61}(1, 0.8, 0.6)$	AbABBBBBBAbAaBabbbbbbaBa
$F_{62}(1, 0.8, 0.6)$	AbABABaBABAbAaBababAbabaBa
$F_{63}(1, 0.8, 0.6)$	AbABBBBBBBAbAaBabbbbbbbaBa
$F_{64}(1, 0.8, 0.6)$	AbABBBBBBBBAbAaBabbbbbbbbaBa
$F_{65}(1, 0.8, 0.6)$	AbaBAbaBabABabAaBABaBAbABabABa
$F_{66}(1, 0.8, 0.6)$	AbABBBBBBBBBAbAaBabbbbbbbbbaBa
$F_{67}(1, 0.8, 0.6)$	AAbAbabAbabAbaBbABaBABaBABaBaa
$F_{68}(1, 0.8, 0.6)$	ABaBabaBabbbaBaBbAbABBBBAbABAbAba
$F_{69}(1, 0.8, 0.6)$	AbaBabABabABAbABBBBAbAaBabbbaBabaBAbaBAbABa
$F_1(1, 0.8, 0.4)$	AbAbBaBa
$F_2(1, 0.8, 0.4)$	AbABbaBa
$F_3(1, 0.8, 0.4)$	AbAbAaBaBa
$F_4(1, 0.8, 0.4)$	AbAbbBBaBa
$F_5(1, 0.8, 0.4)$	AAbAbBaBaa
$F_6(1, 0.8, 0.4)$	BaBBBbbbAb
$F_7(1, 0.8, 0.4)$	AbAbAAaaBaBa
$F_8(1, 0.8, 0.4)$	AAbAbbBBaBaa
$F_9(1, 0.8, 0.4)$	AbAbbbBBBaBa
$F_{10}(1, 0.8, 0.4)$	BaBaaaAAAbAb
$F_{11}(1, 0.8, 0.4)$	AAbAbaABaBaa
$F_{12}(1, 0.8, 0.4)$	AAbAbbbBBBaBaa
$F_{13}(1, 0.8, 0.4)$	AbAbbaBbABBaBa
$F_{14}(1, 0.8, 0.4)$	AbAbbbbBBBBaBa
$F_{15}(1, 0.8, 0.4)$	AABaBaaAAbAbaa
$F_{16}(1, 0.8, 0.4)$	AAAbAbbBBaBaaa
$F_{17}(1, 0.8, 0.4)$	AAbAbAAaaBaBaa
$F_{18}(1, 0.8, 0.4)$	AAAbAbAaBaBaaa
$F_{19}(1, 0.8, 0.4)$	BaBaaaaAAAAbAb
$F_{20}(1, 0.8, 0.4)$	AAAbAbaABaBaaa
$F_{21}(1, 0.8, 0.4)$	AAbAbbbbBBBBaBaa
$F_{22}(1, 0.8, 0.4)$	AbAbAAAAaaaaBaBa
$F_{23}(1, 0.8, 0.4)$	AAAbAbbbBBBaBaaa
$F_{24}(1, 0.8, 0.4)$	AbAbbbbbBBBBBaBa
$F_{25}(1, 0.8, 0.4)$	AAABaBaaAAbAbaaa
$F_{26}(1, 0.8, 0.4)$	BaBaaaaaAAAAAbAb
$F_{27}(1, 0.8, 0.4)$	AbAbabAbAaBaBABaBa
$F_{28}(1, 0.8, 0.4)$	AbAbABAbAaBabaBaBa
$F_{29}(1, 0.8, 0.4)$	AbAbAAAAAaaaaaBaBa
$F_{30}(1, 0.8, 0.4)$	AAAbAbbbbBBBBaBaaa

Table S XI. The free group elements for the periodic three-body orbits

$F_i(m_1, m_2, m_3)$	free group element
$F_{31}(1, 0.8, 0.4)$	AAbAbbbbbBBBBBaBaa
$F_{32}(1, 0.8, 0.4)$	AAAbAbaaaAAABaBaaa
$F_{33}(1, 0.8, 0.4)$	AAAAbAbbbBBBaBaaaa
$F_{34}(1, 0.8, 0.4)$	AbAbbbbbbBBBBBBaBa
$F_{35}(1, 0.8, 0.4)$	AABaBaaaaAAAAbAbaa
$F_{36}(1, 0.8, 0.4)$	AbAbaabAbAaBaBAABaBa
$F_{37}(1, 0.8, 0.4)$	AbABAbAbAAaaBaBabaBa
$F_{38}(1, 0.8, 0.4)$	AAAbAbbbbbBBBBBaBaaa
$F_{39}(1, 0.8, 0.4)$	AAbAbbbbbbBBBBBBaBaa
$F_{40}(1, 0.8, 0.4)$	AAAAbAbbbbbBBBBBaBaaaa
$F_{41}(1, 0.8, 0.4)$	AbAbaaaabAbAaBaBAAAABaBa
$F_{42}(1, 0.8, 0.4)$	AAAAbAbbbbbbBBBBBBaBaaaa
$F_{43}(1, 0.8, 0.4)$	AbaBaBaBaBabAaBABAbAbAbABa
$F_{44}(1, 0.8, 0.4)$	AbAbAbAbAbAbAbAaBaBaBaBaBaBa
$F_1(1, 0.8, 0.2)$	BaAb
$F_2(1, 0.8, 0.2)$	AbAaBa
$F_3(1, 0.8, 0.2)$	BabBAb
$F_4(1, 0.8, 0.2)$	AbAbBaBa
$F_5(1, 0.8, 0.2)$	BabbBBAb
$F_6(1, 0.8, 0.2)$	AbAbAaBaBa
$F_7(1, 0.8, 0.2)$	BaBaaAAbAb
$F_8(1, 0.8, 0.2)$	AbAbbBBaBa
$F_9(1, 0.8, 0.2)$	AbABaAbaBa
$F_{10}(1, 0.8, 0.2)$	BaBaaaAAAbAb
$F_{11}(1, 0.8, 0.2)$	AbAbbbBBBaBa
$F_{12}(1, 0.8, 0.2)$	AAbAbAaBaBaa
$F_{13}(1, 0.8, 0.2)$	AAbAbbBBaBaa
$F_{14}(1, 0.8, 0.2)$	BabbbbBBBBAb
$F_{15}(1, 0.8, 0.2)$	AbAbbbbBBBBaBa
$F_{16}(1, 0.8, 0.2)$	AAbAbAAaaBaBaa
$F_{17}(1, 0.8, 0.2)$	AAAbAbAaBaBaaa
$F_{18}(1, 0.8, 0.2)$	BabbbbbBBBBBAb
$F_{19}(1, 0.8, 0.2)$	AbAbbaBbABBaBa
$F_{20}(1, 0.8, 0.2)$	AAAbAbbBBaBaaa
$F_{21}(1, 0.8, 0.2)$	AbAbAbAbBaBaBaBa
$F_{22}(1, 0.8, 0.2)$	AbAbAAAAaaaaBaBa
$F_{23}(1, 0.8, 0.2)$	AAAbAbAAaaBaBaaa
$F_{24}(1, 0.8, 0.2)$	AAAbAbbbBBBaBaaa
$F_{25}(1, 0.8, 0.2)$	AAbAbAAAAaaaaBaBaa
$F_{26}(1, 0.8, 0.2)$	AbAbAAAAAaaaaaBaBa
$F_{27}(1, 0.8, 0.2)$	AAAAbAbbbbBBBBaBaaaa
$F_{28}(1, 0.8, 0.2)$	AbAbBaBaabAaBAAbAbBaBa
$F_{29}(1, 0.8, 0.2)$	AAAAAbAbbbbBBBBaBaaaaa
$F_{30}(1, 0.8, 0.2)$	AbAbBaBAbABaAbaBabAbBaBa
$F_{31}(1, 0.8, 0.2)$	AbAbBaBaBaBaAbAbAbAbBaBa
$F_{32}(1, 0.8, 0.2)$	AAAAAbAbbbbbBBBBBaBaaaaa
$F_1(0.6, 0.8, 1)$	BaAb
$F_2(0.6, 0.8, 1)$	AbAaBa
$F_3(0.6, 0.8, 1)$	BabBAb
$F_4(0.6, 0.8, 1)$	AbAbBaBa
$F_5(0.6, 0.8, 1)$	BabbBBAb
$F_6(0.6, 0.8, 1)$	AbAbAaBaBa
$F_7(0.6, 0.8, 1)$	BaBaaAAbAb
$F_8(0.6, 0.8, 1)$	AbAbbBBaBa
$F_9(0.6, 0.8, 1)$	AbABaAbaBa
$F_{10}(0.6, 0.8, 1)$	BaBaaaAAAbAb

Table S XII. The free group elements for the periodic three-body orbits

$F_i(m_1, m_2, m_3)$	free group element
$F_{11}(0.6, 0.8, 1)$	AbAbbbBBBBaBa
$F_{12}(0.6, 0.8, 1)$	AAbAbAaBaBaa
$F_{13}(0.6, 0.8, 1)$	AAbAbbBBBaBaa
$F_{14}(0.6, 0.8, 1)$	BabbbbBBBBBAb
$F_{15}(0.6, 0.8, 1)$	AbAbbbbBBBBBaBa
$F_{16}(0.6, 0.8, 1)$	AAbAbAAaaBaBaa
$F_{17}(0.6, 0.8, 1)$	AAAbAbAaBaBaaa
$F_{18}(0.6, 0.8, 1)$	BabbbbbBBBBBBAb
$F_{19}(0.6, 0.8, 1)$	AbAbbaBbABBaBa
$F_{20}(0.6, 0.8, 1)$	AAAbAbbBBaBaaa
$F_{21}(0.6, 0.8, 1)$	AbAbAbAbBaBaBaBa
$F_{22}(0.6, 0.8, 1)$	AbAbAAAAaaaaBaBa
$F_{23}(0.6, 0.8, 1)$	AAAbAbAAaaBaBaaa
$F_{24}(0.6, 0.8, 1)$	AAAbAbbbBBBaBaaa
$F_{25}(0.6, 0.8, 1)$	AAbAbAAAAaaaaBaBaa
$F_{26}(0.6, 0.8, 1)$	AbAbAAAAAaaaaaBaBa
$F_{27}(0.6, 0.8, 1)$	AAAAbAbbbbBBBBBaBaaaa
$F_{28}(0.6, 0.8, 1)$	AbAbBaBaabAaBAAbAbBaBa
$F_{29}(0.6, 0.8, 1)$	AAAAAbAbbbbBBBBBaBaaaaa
$F_{30}(0.6, 0.8, 1)$	AbAbBaBAbABaAbaBabAbBaBa
$F_1(1, 0.412, 0.267)$	BaBaAbAb
$F_2(1, 0.412, 0.267)$	ABaBbAba
$F_3(1, 0.412, 0.267)$	BaBaBbAbAb
$F_4(1, 0.412, 0.267)$	BaBaaAAbAb
$F_5(1, 0.412, 0.267)$	AbAbAaBaBa
$F_6(1, 0.412, 0.267)$	AABaBbAbaa
$F_7(1, 0.412, 0.267)$	AbAbAAaaBaBa
$F_8(1, 0.412, 0.267)$	AAABaBbAbaaa
$F_9(1, 0.412, 0.267)$	AAABaBAabAbaaa
$F_{10}(1, 0.412, 0.267)$	AAAABaBbAbaaaa
$F_{11}(1, 0.412, 0.267)$	AbAbAAAaaaBaBa
$F_{12}(1, 0.412, 0.267)$	BaBAbbAbBaBBabAb
$F_{13}(1, 0.412, 0.267)$	AAAAABaBbAbaaaaa
$F_{14}(1, 0.412, 0.267)$	BaBaBaBaBaBbAbAbAbAbAb
$F_{15}(1, 0.412, 0.267)$	AbAbAbAbAbAbaBbABaBaBaBaBaBaBa
$F_1(0.393, 0.288, 0.662)$	AbaabAaBAABa
$F_2(0.393, 0.288, 0.662)$	AbabaBbABABa
$F_3(0.393, 0.288, 0.662)$	AbaabaBbABAABa
$F_4(0.393, 0.288, 0.662)$	AbabbaBbABBABa
$F_5(0.393, 0.288, 0.662)$	AbaabAAaaBAABa
$F_6(0.393, 0.288, 0.662)$	AbababAaBABABa
$F_7(0.393, 0.288, 0.662)$	AAbabaBbABABaa
$F_8(0.393, 0.288, 0.662)$	AbabaBBbbABABa
$F_9(0.393, 0.288, 0.662)$	AbaaabaBbABAAABa
$F_{10}(0.393, 0.288, 0.662)$	AAbaabaBbABAABaa
$F_{11}(0.393, 0.288, 0.662)$	AbaabaBBbbABAABa
$F_{12}(0.393, 0.288, 0.662)$	AbaabAAAaaaBAABa
$F_{13}(0.393, 0.288, 0.662)$	AbababAAaaBABABa
$F_{14}(0.393, 0.288, 0.662)$	AbabbaBBbbABBABa
$F_{15}(0.393, 0.288, 0.662)$	AbaaaabaBbABAAAABa
$F_{16}(0.393, 0.288, 0.662)$	AbaaabaBBbbABAAABa
$F_{17}(0.393, 0.288, 0.662)$	AbaabaBBBbbbABAABa
$F_{18}(0.393, 0.288, 0.662)$	AbabaBABAababABABa
$F_{19}(0.393, 0.288, 0.662)$	AbabaBABABababAaBABABababABABa

Table S XIII. The free group elements for the periodic three-body orbits

$F_i(m_1, m_2, m_3)$	free group element
$F_1(1, 0.1, 0.01)$	BaBbAb
$F_2(1, 0.1, 0.01)$	ABaBbAba
$F_3(1, 0.1, 0.01)$	BaBaAbAb
$F_4(1, 0.1, 0.01)$	BaBBbbAb
$F_5(1, 0.1, 0.01)$	AbaBAabABa
$F_6(1, 0.1, 0.01)$	AbAbaABaBa
$F_7(1, 0.1, 0.01)$	BaBBBbbbAb
$F_8(1, 0.1, 0.01)$	AbAbaaAABaBa
$F_9(1, 0.1, 0.01)$	AABaBAabAbaa
$F_{10}(1, 0.1, 0.01)$	AbaBAAaabABa
$F_{11}(1, 0.1, 0.01)$	BaBBaBbAbbAb
$F_{12}(1, 0.1, 0.01)$	AAABaBAabAbaaa
$F_{13}(1, 0.1, 0.01)$	AbaBAAAaaabABa
$F_{14}(1, 0.1, 0.01)$	AABaBAAaabAbaa
$F_{15}(1, 0.1, 0.01)$	BaBBaBBbbAbbAb
$F_{16}(1, 0.1, 0.01)$	AAABaBAAaabAbaaa
$F_{17}(1, 0.1, 0.01)$	AbAbaaaaAAAABaBa
$F_{18}(1, 0.1, 0.01)$	AAbaBAAAaaabABaa
$F_{19}(1, 0.1, 0.01)$	AAbAbaaaAAABaBaa
$F_{20}(1, 0.1, 0.01)$	AbaBAAAAaaaabABa
$F_{21}(1, 0.1, 0.01)$	BaBBBaBBbbAbbbAb
$F_{22}(1, 0.1, 0.01)$	AbAbabAbAaBaBABaBa
$F_{23}(1, 0.1, 0.01)$	AAAABaBAAaabAbaaaa
$F_{24}(1, 0.1, 0.01)$	AAABaBAAAaaabAbaaa
$F_{25}(1, 0.1, 0.01)$	AAbaBAAAAaaaabABaa
$F_{26}(1, 0.1, 0.01)$	AbaBAAAAAaaaaabABa
$F_{27}(1, 0.1, 0.01)$	BaBaBBaBaAbAbbAbAb
$F_{28}(1, 0.1, 0.01)$	BaBBabbAbBaBBAbbAb
$F_{29}(1, 0.1, 0.01)$	BaBBaBaBBbbAbAbbAb
$F_{30}(1, 0.1, 0.01)$	BaBBaBBaBbAbbAbbAb
$F_{31}(1, 0.1, 0.01)$	BaBabbAbAbBaBaBBAbAb
$F_{32}(1, 0.1, 0.01)$	BaBabbAbAAaaBaBBAbAb
$F_{33}(1, 0.1, 0.01)$	BaBaBbAbAbBaBaBbAbAb
$F_{34}(1, 0.1, 0.01)$	AAbAbbbAbAbBaBaBBBaBaa
$F_{35}(1, 0.1, 0.01)$	AAAAABaBAAAaaabAbaaaaa
$F_{36}(1, 0.1, 0.01)$	AAAABaBAAAAaaaabAbaaaa
$F_{37}(1, 0.1, 0.01)$	AAAbaBAAAAAaaaaabABaaa
$F_{38}(1, 0.1, 0.01)$	BaBBaBaBaBBbbAbAbAbbAb
$F_{39}(1, 0.1, 0.01)$	BaBBaBBaBaBBbbAbAbbAbbAb
$F_{40}(1, 0.1, 0.01)$	BaBBaBBaBBaBbAbbAbbAbbAb
$F_{41}(1, 0.1, 0.01)$	BaBBaBBBBaBBBbbAbbbbAbbAb
$F_{42}(1, 0.1, 0.01)$	BaBAbABaBabAbaABaBAbAbaBabAb
$F_{43}(1, 0.1, 0.01)$	BaBBaBBaBaBaBBbbAbAbAbbAbbAb
$F_{44}(1, 0.1, 0.01)$	BaBAbAbaBaBaBaBBbbAbAbAbABaBabAb
$F_{45}(1, 0.1, 0.01)$	BaBBaBBaBBaBBaBBaAbbAbbAbbAbbAbbAb
$F_{46}(1, 0.1, 0.01)$	BaBabAbbAbbAbbAbbAaBBaBBaBBaBBaBAbAb
$F_{47}(1, 0.1, 0.01)$	BaBBaBBaBBaBBaBBaBbAbbAbbAbbAbbAbbAb
$F_{48}(1, 0.1, 0.01)$	BaBBaBBaBaBBaBaBBaBBBbAbbAbAbbAbAbbAbbAb