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Observation of two coupled Faraday waves in a vertically vibrating Hele-Shaw cell with one of them oscillating horizontally

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A system of two coupled Faraday waves is experimentally observed at the two interfaces of the three layers of fluids (air, pure ethanol, and silicon oil) in a covered Hele-Shaw cell with periodic vertical vibration. Both the upper and lower Faraday waves are subharmonic, but they coexist in different forms: the upper one vibrates vertically, while the crests of the lower one oscillate horizontally with unchanged wave height, and the troughs of the lower one usually remain in the same place (relative to the basin). Besides, they are strongly coupled: the wave height of the lower Faraday waves is either a linear function (when forcing frequency is fixed) or a parabolic function (when acceleration amplitude is fixed) of that of the upper one with a same wavelength. *Published by AIP Publishing.*
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I. INTRODUCTION

The Faraday waves in a vertically oscillating basin were first observed in 1831[1] and then were analyzed by Benjamin and Ursell,[2] who found that part of these standing waves vibrate vertically with a frequency equal to half of the forcing one of the basin. These waves can organize in many forms, such as stripes, squares, hexagons,[3] and even stars.[4] There are also some other astonishing phenomena that continue to be reported, such as the floating droplets that keep walking and orbiting on the surface of a liquid at a sufficiently high acceleration.[5]

The previous studies usually focused on the motion of the interface between liquid and gas.[6–9] However, the phenomenon which occurs between two liquid films has attracted much attention recently.[10] The condition of the latter problem can be arisen in various ways, such as two layers of fluids of semi-infinite depth in two spatial dimensions (2D),[11] two layers of immiscible fluids of arbitrary depth enclosed between two horizontal plates in two spatial dimensions (2D),[12–14] two immiscible or miscible liquid films of arbitrary depth defined in a three-dimensional container (3D),[15–21] liquid lenses with a free surface partly supported by a bottom layer of liquid,[22,23] and two immiscible layers of fluids with a deformable upper surface.[24]

For two-layer problems in two spatial dimensions, it usually takes into account of the condition of the two immiscible fluids of semi-infinite or arbitrary depth confined in a container. Wright *et al.*[11] numerically studied the fully nonlinear dynamics of two-dimensional Faraday waves at the interface between two inviscid fluids. They simulated the viscous dissipation by adding a phenomenological damping coefficient to the evolution equation. It shows that the surface tension, density ratio, and magnitude of forcing play an important role in determining the motion of the interfacial patterns. The results were compared with the fully nonlinear numerical simulation qualitatively by adopting the phase-filed method; however, there is still a limitation on the Atwood number $A < 0.40$ for the latter work.[12] The linear instability analysis for the interface of two viscous fluids was studied using Floquet theory by Kumar and Tuckerman.[13] They considered the viscous boundary conditions at the interface which the traditional approach always ignored and found that the effect of large viscosity on the wavelength selection is substantial. The following experimental investigation provided a remarkable agreement to their theory[13] and presented the isolation of the causes for the differences.[14]

Although the tremendous progress has been made in two-dimensional theory, the quantitative comparisons between theory and experiments are also necessary in three dimensions as the side-wall boundaries play an important role in determining the motion of the layer. In addition, the new technique makes it possible to measure the spatiotemporal Fourier spectrum of Faraday waves on the interface of two liquids in a three-dimensional closed cell.[15] Especially, the geometry of the container takes on a decisive role in formation of patterns. In an enclosed rectangular domain, a threshold condition for instability of Faraday interfacial waves for a two-layer, weakly viscous system was determined by Hill.[16] The experimental results of interfacial Faraday waves between two immiscible liquids in a cylindrical cell by vertical oscillation were presented. They measured the bifurcation curves and stability diagrams of the system. It indicated that the nonlinear damping varied significantly as a function of filling parameter.[17] By singular perturbation theory, the interfacial wave modes

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in a two-layer liquid-filled cylindrical vessel were found to become more complex, as the density ratio increases from the upper layer to the lower layer.[18] For a system of two immiscible viscous layers in a Hele-Shaw cell oscillated horizontally, a decrease in the viscosity contrast has a stabilizing effect on the Kelvin-Helmholtz instability.[19] Besides, interfacial instability between two miscible liquids subject to vertical oscillation was studied by means of experiments and numerical simulation. It has been shown that the properties of the interfacial waves were much like the case of immiscible fluids.[20,21]

The Faraday instability was then extended to the system of three layers (two layers of liquids and one layer of gas). Considering that the traditional investigation is always constrained by the boundary conditions, Pucci *et al.*[22,23] studied the Faraday instability in floating fluid drops. The steady patterns were observed because the radiation pressure of Faraday waves and capillary response of the lens border gain a balance. They also mentioned that the interface displacement is much weaker than the surface one.[23] Pototsky and Bestehorn[24] investigated the linear Faraday instability of a two-layer liquid film with an entire free upper surface theoretically. Both modes are strongly coupled, and they change several times between the in-phase and anti-phase configuration over one oscillation cycle.

Motivated by earlier studies, we consider an extreme case of Faraday waves in three layers: a rectangular container will be transformed into a Hele-Shaw cell with the width reduced to 2 mm.[25,26] We conduct experiments in such a special container to investigate the formation of the Faraday waves. A system of two coupled Faraday waves at two interfaces of three layers of fluids (i.e., air, pure ethanol and silicone oil) has been observed. The upper Faraday waves (i.e., the interface between air and pure ethanol) vibrate vertically, and the lower Faraday waves (i.e., the interface between the pure ethanol and silicone oil) oscillate horizontally. They coexist and are strongly coupled with a same wavelength. The wave height of the lower Faraday waves is either a linear function (when the forcing frequency is fixed) or a parabolic function (when the acceleration amplitude is fixed) of that of the upper. It also implies that there is a threshold of the ratio of viscosity with the one of the upper fluid fixed within an appropriate range.

The outline of the article is as follows: In Sec. II, the experimental setup is presented. Section III gives the experimental results in three separate parts, namely, (A) the phenomena of coupled Faraday waves, (B) the effect of viscosity of silicone oil, and (C) the influence of forcing frequency and acceleration. Finally, our conclusion and discussion are given in Sec. IV.

II. EXPERIMENTAL SETUP

The experimental setup is shown in (Fig. 1). A Hele-Shaw cell (made of PMMA) with 300 mm length, 2 mm width, and 60 mm depth is filled with two immiscible fluids: the upper one (pure ethanol) has an initial depth d_1 of 4 mm, while the initial depth d_2 of the lower one (silicone oil) is 8 mm. For observation convenience, a very small amount of phenol red has been added in pure ethanol, whose effect on the density and surface tension of the upper fluid is negligible. We use ρ, σ, and μ to denote the density, surface tension, and viscosity, respectively. The upper liquid is pure ethanol (with its physical parameters ρ_1 = 791 kg/m^3, σ_1 = 22.51 mN/m, and μ_1 = 0.0011 Pa s), and we choose six types of silicone oil (from No. 21 to 26) for six sets of experiment. The silicone oil of different numbers corresponds to different parameters, which can be found in Table I. So, in this paper, we use the term "upper Faraday waves" for the interface between the air and the pure ethanol and the term "lower Faraday waves" for the the interface between the pure ethanol and the silicon oil.

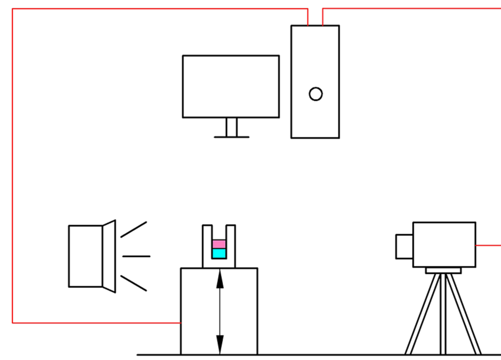

FIG. 1. Schematic illustration of the experimental setup.

The cell is fixed on an electrodynamics vibration generator which can provide a peak force of 500 N and generate a vertical sinusoidal oscillation of acceleration $A(t) = A\cos(2\pi ft)$. The forcing frequency (denoted by f) and the acceleration amplitude (denoted by A) of the shaker are output by a control instrument with the wave form deviation factor <0.3%, and the forcing frequencies resolution is 0.01%. The instrument adjusts the output by measuring the real-time parameters to achieve closed-loop control on the motion of the shaker. A high-speed camera is positioned perpendicular to the front of the cell to record the evolution of the upper interface (gas-liquid) and the lower interface (liquid-liquid). It offers a maximum resolution of 1696 × 1710 pixels (1 pixel = 8 μm) at a speed of 500 fps (1 picture of 2 × 10^{-3} s). The shaker is self-balancing, and other apparatus are adjusted before experiments using a level. A constant temperature (20 ± 0.5 °C) is maintained by air conditioners. Considering volatility of pure ethanol, the cell is covered (i.e., the depth of the air is 48 mm), and the pure ethanol is replaced every half hour.

TABLE I. The parameters of silicone oil.

No.	Viscosity (Pa s)	Density (kg/m^3)	Surface tension (mN/m)	Interfacial tension (mN/m)
21	0.01	900	20	0.73
22	0.05	960	20.8	0.71
23	0.1	963	21	0.71
24	0.35	970	21.1	0.68
25	0.5	970	21.1	0.68
26	1	970	21.2	0.67

III. EXPERIMENTAL RESULTS

A. Coupled Faraday waves

We perform the first experiment with pure ethanol and silicone oil NO.24 mentioned earlier in Table I. When the Hele-Shaw cell vibrates vertically with f = 18 Hz and A = 17 m/s^2, a system of two coupled Faraday waves can be observed. It contains five sub-plots (taken by the high-speed camera) in Fig. 2 (multimedia view), which denote the deformation of the surfaces at t = 0, T/4, T/2, 3T/4, and T in one period, respectively. The solid lines correspond to the Faraday waves at the upper interface, while the grey lines correspond to the lower interface. The details of such waves can be found in the movie of the supplementary material. At the upper interface, there exists a group of standing waves that oscillate *vertically* in a similar way like traditional Faraday waves (but with a few fundamental differences mentioned later), called the upper Faraday waves. At the lower interface, a group of standing waves whose crests oscillate *horizontally* with a nearly constant height and a frequency equal to half of the forcing frequency of the *vertically* vibrating basin can be observed, called the lower Faraday waves. The upper and lower Faraday waves coexist and are strongly coupled, with a same period (denoted by T) and wavelength (denoted by L). At t = 0, the crest of the upper Faraday waves reaches its maximum height, below which there are two adjoining crests of the lower Faraday waves that are in the shortest distance (denoted by $\delta_{\min}$). Thereafter, the crest of the upper Faraday waves falls vertically until it becomes a trough at $t = T/2$, while the above-mentioned two adjoining crests of the lower Faraday waves depart horizontally from each other with almost unchanged height, and their distance (denoted by δ) increases to the maximum (denoted by $\delta_{\max}$) at $t = T/2$. As the time further increases, the trough of the upper Faraday wave moves upwards, but temporarily loses its smoothness at $t = 3T/4$, and then becomes a crest again that reaches its maximum at $t = T$. In the same time, the two adjoining crests of the lower Faraday wave horizontally approach each other with the unchanged height until δ decreases to $\delta_{\min}$ at $t = T$. Note that all troughs of the lower Faraday wave are almost (relative to the Hele-Shaw cell) on the same horizontal line, and the horizontal distance of any two adjoining troughs is equal to the half of the wavelength L of the upper Faraday wave. However, unlike the upper Faraday waves which have symmetry about crests, the lower Faraday waves lose their symmetry about the crests, although both of the upper and lower Faraday waves retain the symmetry about the troughs. Besides, unlike the traditional Faraday waves, the upper Faraday waves temporarily lose their smoothness at around $t = T/4$ and $t = 3T/4$. It implies that the upper and lower Faraday waves strongly interact with each other.

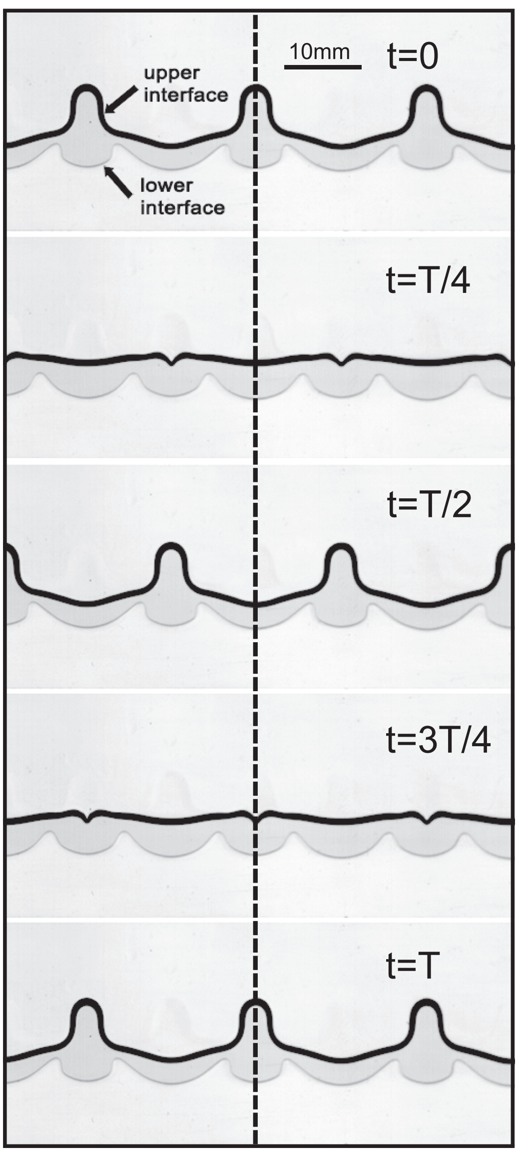

FIG. 2. The system of two coupled Faraday waves observed in two layers of pure ethanol and silicone oil (NO.24)in the case of f = 18 Hz, A = 17 m/s^2, and the vibration amplitude = 1.33 mm, where T denotes the wave period. For more details, please see the corresponding multi-media view. Multimedia view: https://doi.org/10.1063/1.5004452.1

In addition, it is found that, using the same f = 18 Hz and A = 17 m/s^2, we cannot observe any Faraday waves if there only exists silicone oil (NO.24) at a depth of 8 mm in the covered Hele-Shaw cell. Although the surface tension of silicone oil has met the request of wettability of the PMMA cell, the large viscosity inhibits its motion. It also indicates that the lower Faraday waves are induced by the upper one. If we increase the depth of pure ethanol to more than 10 mm, the upper Faraday waves can still be observed, but nothing occurs at the lower interface. It denotes that the thick fluid layer weaken the effect of the deformation at the free surface. This phenomenon strongly suggests that the lower horizontally oscillating Faraday waves are excited by the upper vertically vibrating Faraday waves via the viscous friction on the interface between the two immiscible liquids.

In order to analyze such phenomenon quantitatively, we display the schematic illustration of the upper and lower Faraday waves in Fig. 3(a) and define the following physical parameters: H_1 and H_2 denote the wave height of the upper and lower Faraday waves, L as their wavelength, δ as the horizontal distance between the two adjoining crests of the lower Faraday waves, respectively. In the case of f = 18 Hz and A = 17 m/s^2, the time-dependent variation of δ is shown in Fig. 4(a), which

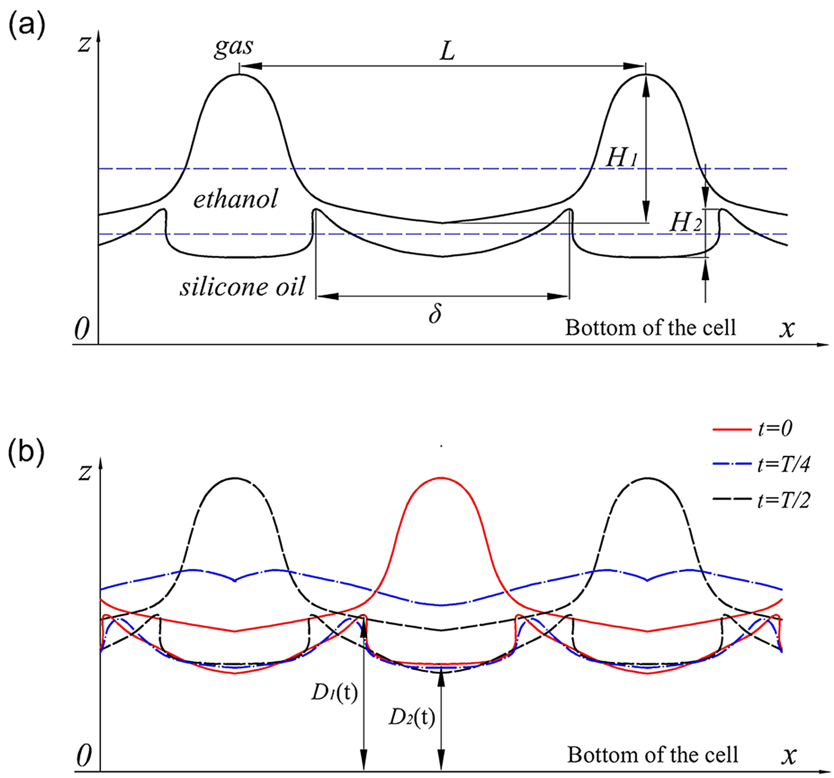

FIG. 3. (a) Schematic illustration of the upper and lower Faraday waves. H_1 and H_2 denote the wave height of the upper and lower Faraday waves, L is their wavelength, δ is the distance of the two crests of the lower Faraday wave, respectively. (b) Wave profiles in different times of a half period. $D_1(t)$ and $D_2(t)$, respectively, denote the vertical distance from a single crest and trough to the bottom of the cell.

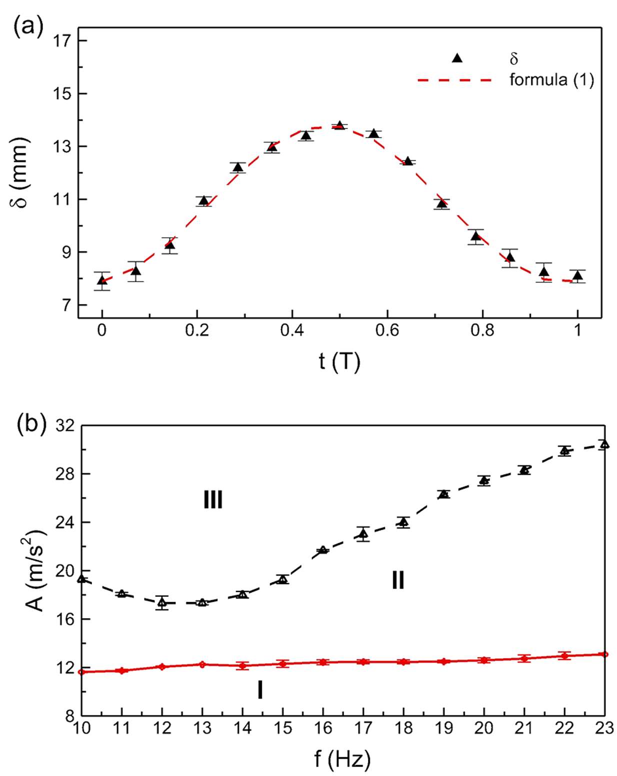

FIG. 4. (a) Variation of $\delta(t)$ in one period in the case of f = 18 Hz, A = 17 m/s^2, and μ_{24} = 0.35 Pa S. (b) Stability diagram for the coupled Faraday waves. Solid line: the onset thresholds of the coupled waves; dashed line: the chaos thresholds. Region I denotes where the free surface is stable; region II denotes where the coupled Faraday waves are stable; region III denotes where the Faraday waves become chaos.

can be fitted by a simple formula,

$$\delta(t) = P + Q\cos(\pi f t + \phi), \tag{1}$$

where $P = 1/2(\delta_{\max} + \delta_{\min})$, $Q = \sqrt{q^2 + C_1^2}$, $\phi = \arctan(q/C_1) - \pi/2$, and $q = 1/2(\delta_{\max} - \delta_{\min})$. In this case, $\delta_{\max} = 13.75$ mm, $\delta_{\min} = 7.89$ mm, and $C_1 = 0.4914$ mm are in a good agreement with the measured data. More detailed analysis based on these physical parameters will be given in Secs. III B and III C accordingly.

B. The effect of viscosity

A family of experiments have been performed with pure ethanol and silicone oil of six viscosities for a fixed forcing frequency $f = 18$ Hz and $A = 15$ m/s^2. This forcing acceleration amplitude is selected considering the thresholds of the waves to become chaos corresponding to different viscosity. We also observe coupled Faraday waves in the cell; however, there are some differences between them. With a very low viscosity ($\mu = 0.01$ Pa s) of silicone oil, the lower waves vibrate vertically with the upper ones, similar to the in-phase coupled pattern mentioned in the study of Pototsky and Bestehorn.[24] With the increase in the viscosity (μ = 0.05 Pa s, 0.1 Pa s), the vertical motion of the lower Faraday waves will decay while the horizontal motion become more obvious, and the Faraday waves perform a mixed pattern with motion in both directions. Once the viscosity is high enough (μ = 0.35 Pa s, 0.5 Pa s, 1 Pa s), the vertical motion will be negligible visually, and the crests of the lower waves move horizontally as Fig. 2 (multimedia view) shows. To investigate the effect of viscosity of silicone oil quantitatively, we define that the horizontal oscillation of

TABLE II. The average standard variance versus viscosity.

No.	Viscosity (Pa S)	SV1 (10^{-3} m)	SV2 (10^{-3} m)
21	0.01	0.672	1.937
22	0.05	0.195	1.429
23	0.1	0.148	0.718
24	0.35	0.091	0.216
25	0.5	0.038	0.159
26	1	0.017	0.051

the lower Faraday waves is the crests and the troughs move horizontally relative to the bottom of the cell, which include two factors: horizontal motion of both the peak of the crests and troughs in one period T. As shown in Fig. 3(b), we measure the vertical distance $D_1(t)$ between a single crest and the bottom of the cell in half period of the waves. It is found that $D_1(t)$ reaches its maximum value at $t = 0$ and minimum at $t = T/2$. The average standard variance (SV1) of them corresponding to different viscosities are shown in Table II. We also measure the vertical distances $D_2(t)$ between the troughs and the bottom of the cell and give the average standard variance (SV2) with the same procedure. Both of them are shown in Table II.

It is obvious that the average standard variance of both condition mentioned decrease with the viscosity, which denotes that the waves tend to oscillate more "horizontally." There is also a connection between SV1 and SV2. We define that the lower Faraday waves can be regarded as 'horizontal' only when SV1 ≤ 0.1. It means the crests and troughs almost move without vertical motion. It is the nonlinear relationship between SV1 and viscosity which determines that there also exists a threshold of the latter in the range of 0.1 Pa s–0.35 Pa s. When the viscosity exceeds the threshold, the lower Faraday waves could be horizontal; otherwise, it is vertical or mixed pattern.

Note that Pototsky and Bestehorn[24] theoretically investigated the linear instability of Faraday waves of the three-layer fluids (air and two immiscible liquids) in a *three-dimensional* domain. They gained the coupled 2 F waves that vibrate vertically in two patterns. Even the patterns we observed with a lower viscosity of silicone oil are qualitatively similar to the in-phase waves they mentioned, but, the horizontal pattern with a higher viscosity is very different from their results. Unlike their theoretical investigation, our physical experiments are related to the Faraday waves *in a Hele-Shaw cell* by means of physical parameters quite different from theirs: the horizontal Faraday waves is observed with the minimum ratio of

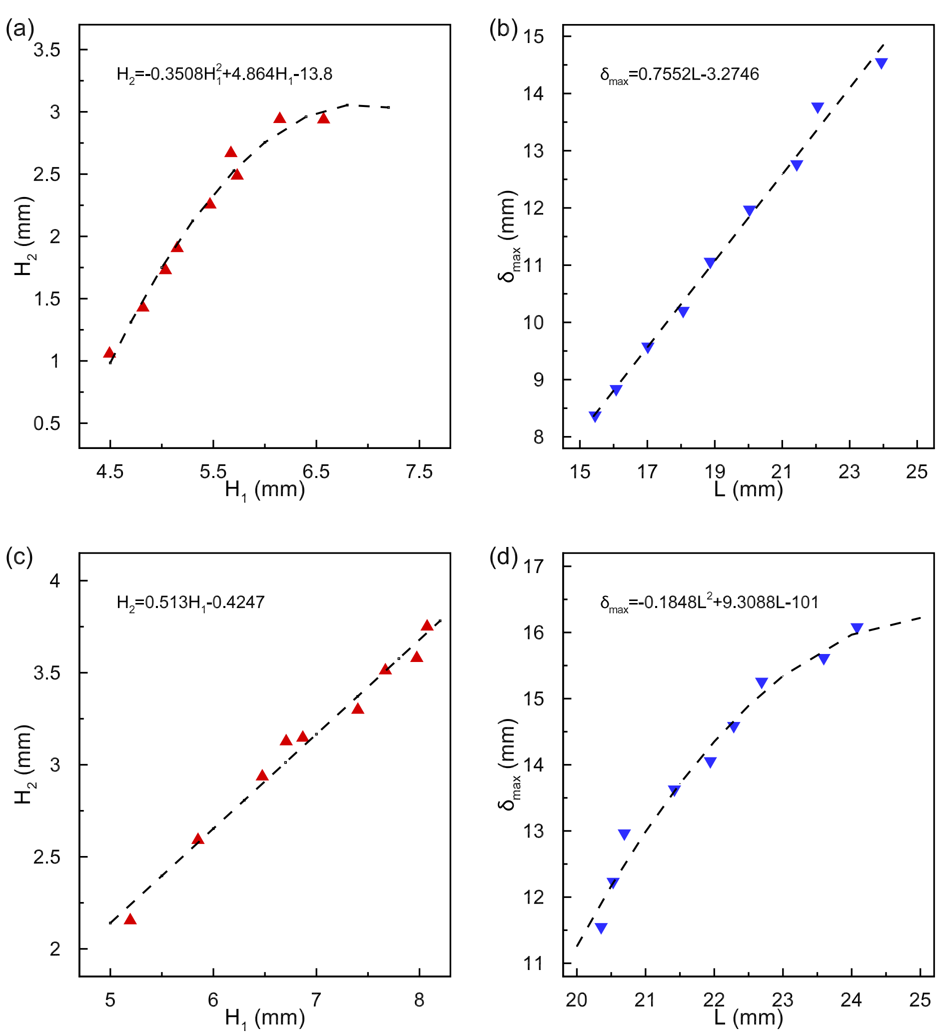

FIG. 5. Relations between H_1 and H_2, L and δ_{max} of the coupled 2 F waves. [(a) and (b)] In the case of $A = 15$ m/s^2 and 15 Hz $\leq f \leq$ 23 Hz; [(c) and (d)] in the case of $f = 18$ Hz and 14 m/s^2 $\leq A \leq$ 22 m/s^2; dashed-line: the fitting formulas.

the viscosity $\mu_2/\mu_1 \approx 318.2$ in our experiment, which is about 22 times larger than that considered in their work.[24] It implies that there is always a threshold of the ratio of viscosity with the one of the upper fluid fixed within an appropriate range.

Although, the surface tension has effect on the formation of these waves, we tend to believe that due to the geometry of PMMA cell, other parameters play more important roles to the generation of the Faraday waves as long as the surface tension meet the request of the wettability.

C. The effect of forcing frequency and acceleration

With f fixed as 18 Hz, let A increase slowly with an increment of 0.1 m/s^2 to see excitation and development of the waves. The system of the two coupled Faraday waves can be observed within an approximate region of the acceleration amplitude $A_{\min}(12.467 \pm 0.153\ \text{m/s}^2) \le A \le A_{\max}(23.967 \pm 0.451\ \text{m/s}^2)$. When $A < A_{\min}$, no interfacial waves were observed at all. When $A > A_{\max}$, the interfacial waves at the upper and lower interfaces are disordered. In the cases of f = 23 Hz and f = 10 Hz, such kinds of two coupled Faraday waves are always observed but with different upper and lower thresholds of A. It is found that there exist the corresponding upper and lower thresholds of A for a given f, as shown in Fig. 4(b). Note that while the lower threshold of A increases with f very slowly, the upper threshold decreases slightly before rising rapidly, making a trough at frequency around 12 Hz. The upper threshold at 10 Hz is slightly higher than that at 12 Hz. The reason for such behavior is due to the increase in the friction. Considering the geometry of the Hele-Shaw cell, the contact area of the Faraday waves at forcing frequency of 10 Hz is significantly larger than it at 12 Hz, and the increase in friction promotes the stability of the Faraday instability.

In the case of the fixed acceleration amplitude A = 15 m/s^2 with the different f in the range of 15 Hz $\le f \le$ 23 Hz, the wave height of the lower Faraday wave, or H_2, is a parabolic function of the wave height of the upper one (H_1), while $\delta_{\max}$ remains a linear relationship with respect to the wavelength L, as shown in Figs. 5(a) and 5(b). For the fixed f = 18 Hz but different A in the region of 14 m/s$^2 \le A \le$ 22 m/s^2, the wave heights of the lower Faraday waves H_2 have a linear relationship with the wave height H_1 of the upper one, but $\delta_{\max}$ is a parabolic function of the wavelength L, respectively, as shown in Figs. 5(c) and 5(d). These phenomena reveal the close relationship and strong coupling between the upper and lower Faraday waves.

Note that the upper Faraday waves look like the traditional Faraday waves at the interface of two immiscible fluids only (such as water and air). For comparison, we measured the traditional Faraday waves at the interface of the air and pure ethanol with the same depth of 4 mm (but *without* silicon oil) in the same Hele-Shaw cell.[25,26]

In the case of the fixed A = 15 m/s^2 with the different f in the region of 15 Hz $\le f \le$ 23 Hz, both of the wave height H_1 and wavelength L of the upper Faraday wave decrease with the increase in the frequency f, as shown in Figs. 6(a) and 6(b).

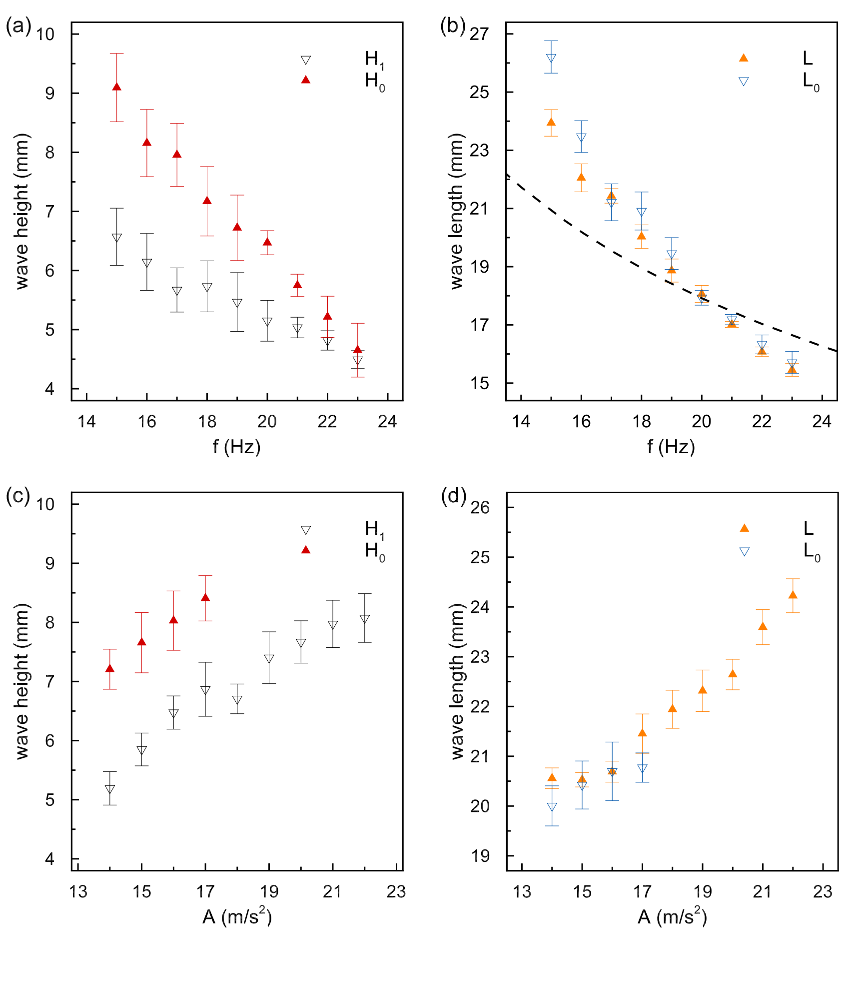

FIG. 6. Comparison of the upper Faraday wave and the traditional Faraday wave of pure ethanol with the same physical parameters (but without the layer of silicon oil below), where H and L denote wave height and wavelength of the upper Faraday wave, H_0 and L_0 denote those of the traditional one, respectively. [(a) and (b)] In the case of A = 15 m/s^2 and 15 Hz $\le f \le$ 23 Hz, the black solid line denotes the dispersion relation; [(c) and (d)] in the case of f = 18 Hz and 14 m/s$^2 \le A \le$ 22 m/s^2.

It is also found that the wave height H_1 of the upper Faraday wave is always smaller than the wave height H_0 of the traditional one, but the wavelength L of the upper Faraday wave is almost the same as that of the traditional one. This is easy to understand, since the upper layer liquid (pure ethanol) transfers some kinetic energy to the lower one (silicon oil) via viscous friction at their interface.

Besides, we compare the dispersion relation of the coupled waves with the dispersion curve mentioned in the work of Pototsky and Bestehorn.[24] For a liquid film with thickness h, density ρ, and surface tension σ, the curve shown in Fig. 6(b) is given by

$$\Omega(k)^2 = (gk + \frac{\sigma k^3}{\rho})\tanh(kh), \qquad (2)$$

where $\Omega = \pi f t$, $k = 1/L$, $\sigma = \sigma_1 = 22.51$ mN/m, $\rho = \rho_1 = 791$ kg/m^3, $g = 9.81$ m/s^2, and $h = d_1 + d_2 = 12$ mm. Although the curves agree with their results very well,[24] the curve is significantly different from our experiment results. The direction of them is distinct from each other, and the only one point of intersection is at f of 20 Hz. We think that it is due to the influence of the geometry of the Hele-Shaw cell and the traditional theories may be unable to predict the exact characteristics of these waves.

For the fixed frequency $f = 18$ Hz with different A in the region 14 m/s^2 $\leq A \leq$ 22 m/s^2, both of the wave height H_1 and wavelength L of the upper Faraday wave increase with the increase of A, as shown in Figs. 6(c) and 6(d). However, it is interesting that the traditional Faraday wave becomes disordered when $A > 17$ m/s^2, but the system of the two coupled Faraday waves exists even for the amplitude A up to 22 m/s^2. It means that, for a given f, the upper threshold of A for the coexistence of the two coupled Faraday waves is larger than that for only one traditional Faraday wave. It indicates that the system of the two coupled Faraday waves is stable even for a large A that corresponds to a stronger nonlinearity. It is easy to expect that, for a given frequency f, more kinetic energy is needed to excite the system of the two coupled Faraday waves than the only one traditional Faraday waves.

In addition, the wavelength L of the upper Faraday waves is almost the same as L_0 of the traditional one, although its wave height H_1 is always smaller than H_0. Furthermore, unlike the traditional Faraday waves, the upper Faraday waves temporarily lose their smoothness at around $t = T/4$ and $3T/4$, as shown in Fig. 2 (multimedia view). All of these indicate that the upper Faraday waves are fundamentally different from the traditional one, although both of them are vertically vibrating waves. Finally, it should be emphasized that the upper and lower Faraday waves coexist and are strongly coupled: neither of them can exist alone.

IV. CONCLUSION

In conclusion, we experimentally observed a system of two coupled Faraday waves at two interfaces of three layers of fluids (air, pure ethanol, and silicon oil) in a covered Hele-Shaw cell with periodic vertical vibration. The upper Faraday waves vibrate vertically, and the lower Faraday waves oscillate horizontally. They coexist and are strongly coupled. With the viscosity of the upper fluid fixed within an appropriate range, there is a threshold of the viscosity of the lower one for them to be observed. This system of two coupled Faraday waves has never been reported, to the best of our knowledge. So, it fleshes out the picture of Faraday waves as a type of vertical standing waves. They also bring us some new challenges in theoretical analyses and numerical simulations.

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