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Pattern transition of two-dimensional Faraday waves at an extremely shallow depth†

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In this paper, we experimentally investigate the pattern transition of two-dimensional Faraday waves at an extremely shallow depth in a Hele-Shaw cell. Several patterns of Faraday waves are observed, which have some significant differences in wave profile, wave height and wave length. It is found that, in a wide range of the forcing frequency f, there always exists a region of the acceleration amplitude A, in which there exist the so-called hysteretic jumps between different patterns of Faraday waves. All of these experimental observations could enrich our knowledges about the Faraday waves and would be helpful to the further theoretical studies on the related topic in future.

Faraday waves, extremely shallow water, experimental observation

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1 Introdcution

The Faraday waves were first observed through vertical oscillation [1]. It is remarked that for some combinations of the driving frequency and forcing acceleration amplitude, Faraday waves appear at the free surface when with others it remains plane [2]. The excitation of Faraday waves was analysed in details and plenty of patterns were observed, such as stripes, squares, hexagons and stars [3]. It is very interesting that the transitions between triangles and hexagons can be enforced by perturbing with another frequency [4]. The instability of these patterns in deep layer has been explained in terms of a systematic nonlinear theory [5] in quantitative agreement with experiments [6]. Binks et al. [7] found a cascade of surface wave patterns in shallow layer and reported that the symmetry of standing wave patterns on the surface of three dimensional fluid layer through vertical oscillation depends on the depth of the layer. Their experiments concerning the excitation of subharmonic surface waves also agree with the prediction of theory [8]. Westra devise a simple model that can explain the dependence by means of wave resonance theories [9]. The Faraday instability in shallow layer was also studied with flexible boundaries [10]. The threshold and mode selection of Faraday wave were studied for a frequency range including subharmonic and super-harmonic modes in small cylinders [11]. However, all of these theories were formulated for Faraday waves in three dimensional basin. Recently, a new family of two-dimensional Faraday waves in an extremely shallow layer with a few different patterns was observed in a Hele-Shaw cell [12]. They have some rather different characteristics from the traditional three-dimensional patterns.

In this paper, we report some experimental observations about the transition of these patterns. The influence of phys-

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ical parameters (including forcing frequency, forcing acceleration amplitude and the depth of layer) on the patterns is studied. It is found that the transition is determined mainly by the acceleration amplitude and the layer depth. Especially, in a wide range of the forcing frequency, there always exists a region of the acceleration amplitude, in which there exist the so-called hysteretic jumps between different patterns of Faraday waves.

2 Experimental setup

First, let us briefly describe the experimental set up. A Hele-Shaw cell with a length of 300 mm and a width of 2 mm is fixed on a vertically vibrating shaker. The cell is sealed and filled with pure ethanol to a height (expressed by D) of 2 mm, with the density $\rho = 791$ kg/m^3 and dynamic viscosity $\mu = 1.096 \times 10^{-3}$ Pa S. The temperature is nearly 25°C. The cell is vibrated in the vertical direction with a purely sinusoidal motion $z = A\sin(2\pi f t)$, where A and f denote the forcing frequency and acceleration amplitude of the shaker, respectively, which are given by a closed-loop control system. A high-speed camera (500 fps) is placed perpendicularly to the cell for the investigation of the free surface transients.

3 Results

First, let us fix the layer depth D at 2 mm and the forcing frequency f at 18 Hz, but increase the acceleration amplitude A gradually. The Faraday waves in the extremely shallow depth are observed when the acceleration amplitude A exceeds the lower threshold value, as shown in Figure 1(a), which is defined as pattern-1. The pattern-1 Faraday wave looks like the normal ones.

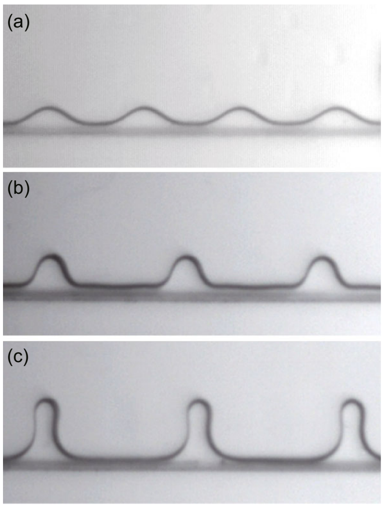

Figure 1 Pattern profiles observed in case of the forcing frequency f = 18 Hz. (a) Pattern-1 with the acceleration amplitude A = 17 m/s^2 and the layer depth D = 2 mm; (b) pattern-2 with the acceleration amplitude A = 19.8 m/s^2 and the layer depth D = 2 mm; (c) pattern-3 with the acceleration amplitude A = 22.62 m/s^2 and the layer depth D = 2 mm.

Continue to increase the acceleration amplitude A, the pattern-2 Faraday wave appears, as shown in Figure 1(b), which is quite different from the pattern-1: the trough becomes a long, almost straight line segment, and most of the liquid is gathered at crest so that the wave height increases greatly, corresponding to a high nonlinearity. Keep on increasing the acceleration amplitude A and we can get the pattern-3 Faraday wave, as shown in Figure 1(c). Note that its trough almost touch the bottom and its crest looks like an inverted droplet with a quite large height, corresponding to a extremely high nonlinearity.

There exist significant differences between these patterns, not only in the wave profile but also in the wave height h and wave length L, as shown in Figures 2(a) and (b). For a fixed layer depth D, the wave height increase with the acceleration amplitude, as shown in Figure 2(a). For pattern-1, the wave height varies in a steadily certain range, but it has demonstrated phenomenal growth with the forcing acceleration since it changes to pattern-2. When the acceleration amplitude A exceeds the threshold of pattern-3, the wave height increases almost twice. At the same time, the wave length L of these patterns follow the similar way but not so rapidly, it grows slowly with the forcing acceleration, but it always shifts dramatically over the thresholds, which are shown in the way of black lines in Figures 2(a) and (b).

There are two different ways to find out the transition

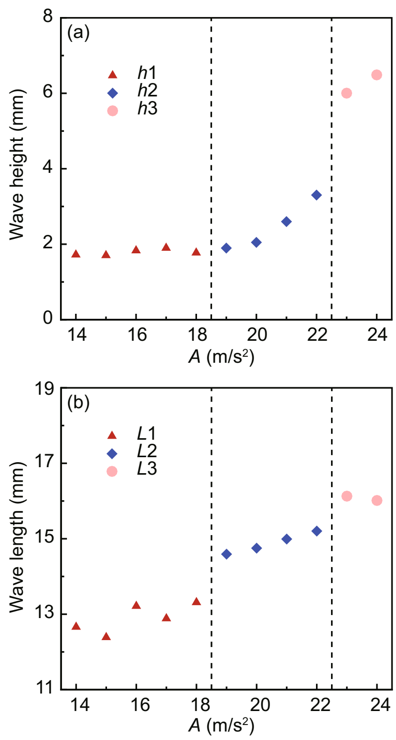

Figure 2 (Color online) (a) Wave heights (h) of Faraday waves at the forcing frequency f =18 Hz and the depth D =2 mm; (b) wave lengths (L) of Faraday waves in the same case. Here, $h1$ and $L1$ denote the pattern-1, $h2$ and $L2$ the pattern-2, $h3$ and $L3$ the pattern 3. The black dashed lines in both figures denote the thresholds of the patterns.

thresholds of different patterns of the Faraday waves in the extremely shallow depth. One is to increase the acceleration amplitude A step by step. The other is to decrease it step by step from a large enough value of A. It is found that the two ways give different values of transition threshold, and the latter one is always a bit smaller, as shown in Figure 3. It suggest that, in some cases, different patterns could appear, dependent up how we vary the forcing acceleration amplitude A for a fixed frequency f. This is the so-called "hysteretic jump", which is widely found in many nonlinear oscillation systems.

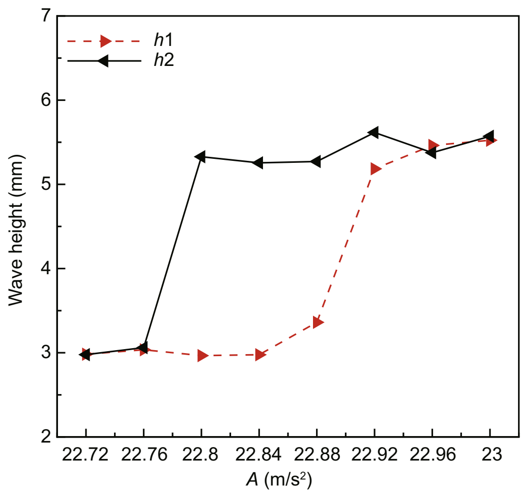

Figure 3 (Color online) The wave height (h) versus the acceleration amplitude A at the forcing frequency $f = 18$ Hz and the layer depth $D = 2$ mm. Here, $h1$ denotes the wave height got by increasing the forcing acceleration step by step, $h2$ denotes the wave height got by decreasing the forcing acceleration step by step.

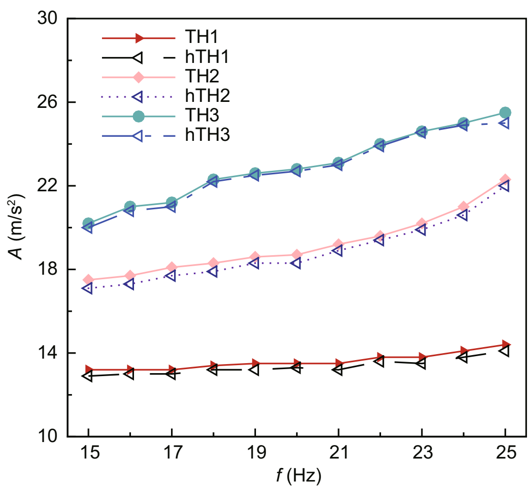

Figure 4 (Color online) The thresholds and hysteretic thresholds of the acceleration amplitude A versus the forcing frequency f at the layer depth $D = 2$ mm. Here, TH1 denotes the threshold of the pattern-1 and hTH1 the hysteretic threshold of the same pattern, TH2 and hTH2 denote those of the pattern-2, TH3 and hTH3 denote those of the pattern-3.

To make a further investigate of the "hysteretic jump", more experiments have been done in a certain range of forcing frequency (15-25 Hz), as shown in Figure 4. It is interesting that such kind of hysteretic jump between different patterns of the Faraday waves in the extremely shallow depth widely exists. Because the thresholds are very sensitive to the depth, we only give the average value. It suggest that the hysteretic state is always very narrow and become even smaller with the forcing acceleration.

4 Conclusion

In this paper, we experimentally investigate the pattern transition of Faraday waves at an extremely shallow depth in a Hele-Shaw cell. Several patterns of Faraday waves are observed, which have some significant differences in wave profile, height and length. It is found that, in a wide range of the forcing frequency f, there always exists a region of the acceleration amplitude A, in which the so-called hysteretic jumps between different patterns of Faraday waves are observed. All of these experimental observations could enrich our knowledges about the Faraday waves and would be helpful to the further theoretical studies on the related topic in future.

Note that the Faraday waves are observed in the extremely shallow depth. This provides us a new challenge for numerical simulations. Possibly, some new numerical methods with rather high accuracy, such as the so-called Clean Numerical Simulation (CNS) [13, 14], might be helpful to investigate these interesting phenomena of Faraday waves in the extremely shallow depth.

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