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SERIES SOLUTIONS OF UNSTEADY MHD FLOW OF A NON-NEWTONIAN FLUID NEAR THE FORWARD STAGNATION POINT

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Unsteady flow of a viscous incompressible electrically conducting non-Newtonian power-law fluid near the forward stagnation point of a two-dimensional body in the presence of a magnetic field is studied by means of an analytic technique, namely, the homotopy analysis method. Accurate analytic approximations are obtained, which are uniformly valid for all dimensionless time in the whole region $0 \leq \eta < \infty$. Effects of integral power-law index of the non-Newtonian fluids for $\kappa = 1, 2, 3$ and magnetic number $M \leq 10$ on the flow are considered. To the best of authors' knowledge, such kinds of analytic solutions have been never reported.

Introduction. Non-Newtonian fluids are very important fluids, which are widely used in industry. Most particulate slurries (china clay and coal in water, sewage sludge, etc.), multiphase mixtures (oil-water emulsions, gas-liquid dispersions, such as froths and foams, butter) are non-Newtonian fluids. Further examples, including a variety of non-Newtonian characteristics, include pharmaceutical formulations, cosmetics and toiletries, paints, synthetic lubricants, biological fluids (blood, synovial fluid, saliva), and foodstuffs (jams, jellies, soups, marmalades). Indeed, behaviours of the non-Newtonian fluids are so widespread that it would be no exaggeration to say that the Newtonian fluid behaviours are an exception rather than the rule. Since the non-Newtonian fluids have more complicated equations that relate the shear stresses to the velocity field than the Newtonian fluids have, additional factors must be considered in examining various fluid mechanics (see, Irvine and Karni [1]).

Several models have been proposed to describe the non-Newtonian behaviour of fluids. Among these models, which are known to follow the empirical Ostwaald-de Waele model, the so-called power-law model, in which the shear stress varies according to a power function of the strain rate, has received wide acceptance. Boundary layer assumptions were successfully applied to this model and much work has been done on it. Schowalter [2] and Acrivos *et al.* [3] initially theoretically analyzed the steady boundary layer flow of incompressible non-Newtonian power-law fluids and found the existence of the similarity solutions. Kim *et al.* [4] made a detailed analysis to obtain possible similarity solutions of the steady boundary layer equations of a non-Newtonian fluid. Yükselen and Erim [5] considered the curvature effects on the boundary layer of a non-Newtonian fluid. Thompson and Snyder [6], and Kim and Eraslan [7] investigated the effect of wall mass injection on the flow of a non-Newtonian power-law fluid over a flat plate, near a stagnation point and past a wedge. Akçay and Yükselen [8] analyzed the boundary layer of a non-Newtonian fluid flow with fluid injection on a semi-infinite flat plate, which moves with a constant velocity in the direction opposite to that of the uniform mainstream. All these kinds of problems have been studied theoretically, numerically and experimentally, by many researchers such as Djukic [9], Helmy [10–12], Ariel [13]. Several excellent books and review papers, summarizing the state-of-the-art available in the literature, testify to the maturity of the field of non-Newtonian fluids, e.g., Astarita and Marrucci [14], Darby [15], Schowalter [16], Tanner [17], Bird *et al.* [18], Crochet *et al.* [19], Andersson and Irgens [20], Shenoy and Mashelkar [21], and Ghosh *et al.* [22].

Most of the previous theoretical and experimental studies of the non-Newtonian fluids have considered steady-state boundary layer flow problems, but little work has been done on unsteady boundary layer flows of the non-Newtonian fluids. Xu and Liao [23] investigated the unsteady magnetohydrodynamic viscous flows of non-Newtonian fluids caused by an impulsively stretching plate. Xu, Liao and Pop [24] considered the unsteady flow of a non-Newtonian power-law fluid in the forward stagnation region of a body and obtained accurate analytic solutions by applying the homotopy analysis method.

The objective of this paper is to study analytically the problem of unsteady flow of a viscous incompressible electrically conducting non-Newtonian power-law fluid near the forward stagnation region of a two-dimensional body subject to a magnetic field by the homotopy analysis method [26–29]. To the best of authors' knowledge, no one has ever reported such kind of analytic solutions, which are valid for all time $0 \leq \tau < \infty$ in the whole region $0 \leq \eta < \infty$.

1. Mathematical description. Consider an unsteady flow of a viscous incompressible electrically conducting non- Newtonian power-law fluid in the vicinity of the forward stagnation of a two-dimensional body subject to a constant magnetic field. The magnetic field is applied normal to the surface of the body and fixed relative to the fluid. It is assumed that the magnetic Reynolds number $\mathrm{Re} = \mu_0 \sigma V L \ll 1$, where μ_0 is the magnetic permeability, σ is the electric conductivity and V and L are the characteristic velocity and length, respectively. It is also assumed that when $t < 0$, the surface of the body and the fluid are at rest. Then, at time $t = 0$, the external stream is set into an impulsive motion from rest with a velocity $u_\mathrm{e}(x) = ax$, where x is the Cartesian coordinate measured along the surface of the body started from the stagnation point and $a > 0$ is a constant. Under these conditions the governing equations for the unsteady boundary layer flows are

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \,, \tag{1}$$

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = u_\mathrm{e}\frac{\mathrm{d}u_\mathrm{e}}{\mathrm{d}x} + \frac{K}{\rho}\frac{\partial}{\partial y}\Big(\frac{\partial u}{\partial y}\Big)^{\kappa} - \frac{\sigma B_0^2}{\rho}\big(u - u_\mathrm{e}\big) \,, \tag{2}$$

where y is the Cartesian coordinate measured normal to the surface of the body, u and v are the velocity components along the x- and y-axes, K, ρ, σ and B_0 are the viscosity, density, electric conductivity and magnetic field, respectively. κ is the index in the power-law variation of the shear stress of a non-Newtonian fluid with $\kappa < 1$ for the pseudoplastic fluid, $\kappa > 1$ for the dilatant fluid and $\kappa = 1$ for the Newtonian fluid, respectively. The corresponding initial and boundary conditions are

$$t < 0: \quad u = v = 0, \quad \text{for all points} \quad (x, y) \,, \tag{3a}$$

$$t \geq 0: \quad u = v = 0, \quad \text{on} \quad y = 0, \quad x \geq 0 \,, \tag{3b}$$

$$t \geq 0: \quad u = u_{\rm e}(x) = ax, \quad \text{as} \quad y \to \infty \,. \tag{3c}$$

Let ψ denote the stream function. Following Williams and Rhyne [30], Nazar *et al.* [31], Liao [32] and Xu and Liao [23], we use the following similarity transformations

$$\begin{aligned} \psi &= (\rho a^{1-2\kappa}/K)^{-1/(\kappa+1)} x^{2\kappa/(\kappa+1)} \xi^{1/(\kappa+1)} f(\eta,\xi) \,, \\ \eta &= (\rho a^{2-\kappa}/K)^{-1/(\kappa+1)} x^{(1-\kappa)/(\kappa+1)} \xi^{-1/(\kappa+1)} y \,, \\ \xi &= 1 - \exp(-\tau), \quad \tau = at \,. \end{aligned} \tag{4}$$

Then, the partial differential equations (1)–(2) are reduced to one partial differential equation.

$$\begin{aligned} (1-\xi)\left(\frac{1}{\kappa+1}\eta\frac{\partial^2 f}{\partial\eta^2} - \xi\frac{\partial^2 f}{\partial\eta\partial\xi}\right) + \kappa\left(\frac{\partial^2 f}{\partial\eta^2}\right)^{\kappa-1}\frac{\partial^3 f}{\partial\eta^3} + \\ + \xi\left[\frac{2\kappa}{\kappa+1}f\frac{\partial^2 f}{\partial\eta^2} + 1 - \Big(\frac{\partial f}{\partial\eta}\Big)^2 + M\Big(1-\frac{\partial f}{\partial\eta}\Big)\right] = 0 \,, \end{aligned} \tag{5}$$

subject to the boundary conditions

$$f(0,\xi) = 0, \qquad \left.\frac{\partial f(\eta,\xi)}{\partial\eta}\right|_{\eta=0} = 0, \qquad \left.\frac{\partial f(\eta,\xi)}{\partial\eta}\right|_{\eta=0} = 1 \,. \tag{6}$$

At $\xi = 0$, corresponding to $\tau = 0$, Eq. (5) reduces to

$$\frac{1}{\kappa+1}\eta\frac{\partial^2 f}{\partial\eta^2} + \kappa\Big(\frac{\partial^2 f}{\partial\eta^2}\Big)^{\kappa-1}\frac{\partial^3 f}{\partial\eta^3} = 0 \,, \tag{7}$$

subject to the boundary conditions

$$f(0,0) = \left.\frac{\partial f(\eta,\xi)}{\partial\eta}\right|_{\eta=0,\,\xi=0} = 0, \qquad \left.\frac{\partial f(\eta,\xi)}{\partial\eta}\right|_{\eta=\infty,\,\xi=0} = 1 \,. \tag{8}$$

At $\xi = 1$, corresponding to $\tau = \infty$ for steady-state flows, Eq. (5) reduces to

$$\kappa\Big(\frac{\partial^2 f}{\partial\eta^2}\Big)^{\kappa-1}\frac{\partial^3 f}{\partial\eta^3} + \frac{2\kappa}{\kappa+1}f\frac{\partial^2 f}{\partial\eta^2} + 1 - \Big(\frac{\partial f}{\partial\eta}\Big)^2 + M\Big(1-\frac{\partial f}{\partial\eta}\Big) = 0 \,, \tag{9}$$

subject to the boundary conditions

$$f(0,1) = \left.\frac{\partial f(\eta,\xi)}{\partial\eta}\right|_{\eta=0,\,\xi=1} = 0, \qquad \left.\frac{\partial f(\eta,\xi)}{\partial\eta}\right|_{\eta=\infty,\,\xi=1} = 1 \,, \tag{10}$$

where $M = \sigma B^2/(\rho a)$ is the magnetic parameter.

Note that different from the corresponding unsteady flow of the Newtonian fluid, even the equation at $\xi = 0$ is nonlinear for the non-Newtonian fluid. Thus, the nonlinearity under consideration is rather strong.

The quantity with physical interest is the local skin friction coefficient, $C_{\rm f}$, defined by

$$C_{\rm f}({\rm Re}_x)^{1/(\kappa+1)} = 2\xi^{-1\kappa/(\kappa+1)}\left[\frac{\partial^2 f}{\partial\eta^2}(\xi,0)\right]^{\kappa} \,, \tag{11}$$

where ${\rm Re}_x = \rho x(u_{\rm e})^{2-\kappa}/K$ is the local Reynolds number.

2. Homotopy analysis solution. In this section, we solve Eq. (5) analytically by the homotopy analysis method. From the boundary condition (6) it is obvious that $f(\eta,\xi)$ can be expressed by a set of base functions

$$\left\{\xi^j\eta^\kappa \exp(-n\eta) \,|\, j\geq 0,\ k\geq 0,\ n\geq 0\right\} \tag{12}$$

in the form

$$f(\eta,\xi)=\sum_{j=0}^{+\infty}\sum_{k=0}^{+\infty}\sum_{n=0}^{+\infty} a_{k,n}^j \xi^j\eta^k \exp(-n\eta)\ , \tag{13}$$

where $a_{k,n}^j$ is the coefficient to be determined. Thus, all approximations of $f(\eta,\xi)$ must obey the above expressions: this is so important in the frame of the homotopy analysis method that they should be taken as a rule called *the Rule of Solution Expression* for $f(\eta,\xi)$. From the boundary conditions (6), and considering the *Rule of Solution Expression* denoted by (13), it is straightforward to choose

$$f_0(\eta,\xi)=\eta+\exp(-\eta)-1\ , \tag{14}$$

as an initial approximation of $f(\eta,\xi)$. Similarly, it is natural to choose

$$\mathcal{L}_f\left[\Phi(\xi,\eta;q)\right]=\exp(\eta)\left[\frac{\partial^3\Phi}{\partial\eta^3}-\frac{\partial\Phi}{\partial\eta}\right] \tag{15}$$

as an auxiliary linear operator, which has the following property

$$\mathcal{L}_f\left[C_1\exp(-\eta)+C_2\exp(\eta)+C_3\right]=0\ , \tag{16}$$

where C_1, C_2 and C_3 are the constants. From (5), we define a nonlinear operator

$$\begin{aligned}\mathcal{N}_f\left[\Phi(\xi,\eta;q)\right]=(1-\xi)\left(\frac{1}{\kappa+1}\eta\frac{\partial^2\Phi}{\partial\eta^2}-\xi\frac{\partial^2\Phi}{\partial\eta\partial\xi}\right)+\kappa\left(\frac{\partial^2\Phi}{\partial\eta^2}\right)^{\kappa-1}\frac{\partial^3\Phi}{\partial\eta^3}+\\ +\xi\left[\frac{2\kappa}{\kappa+1}\Phi\frac{\partial^2\Phi}{\partial\eta^2}+1-\left(\frac{\partial\Phi}{\partial\eta}\right)^2+M\left(1-\frac{\partial\Phi}{\partial\eta}\right)\right],\end{aligned} \tag{17}$$

where q is an embedding parameter, $\Phi(\xi,\eta;q)$ denotes mappings of $f(\eta,\xi)$. Let $\hbar_f$ denote the auxiliary non-zero parameter. We construct the so-called zero-order deformation equation

$$(1-q)\mathcal{L}_f\left[\Phi(\xi,\eta;q)-f_0(\eta,\xi)\right]=q\hbar_f\mathcal{N}_f\left[\Phi(\xi,\eta;q)\right]\ , \tag{18}$$

subject to the boundary conditions

$$\Phi(0,\xi;q)=\left.\frac{\partial\Phi(\eta,\xi;q)}{\partial\eta}\right|_{\eta=0}=0,\qquad \left.\frac{\partial\Phi(\eta,\xi;q)}{\partial\eta}\right|_{\eta=+\infty}=1\ . \tag{19}$$

Obviously, when $q=0$ and $q=1$, the above zero-order deformation equation (18) has the solutions

$$\Phi(\eta,\xi;0)=f_0(\eta,\xi) \tag{20a}$$

and

$$\Phi(\eta,\xi;1)=f(\eta,\xi)\ , \tag{20b}$$

respectively. Thus, as q increases from 0 to 1, $\Phi(\eta,\xi;q)$ varies (or deforms) from the known initial guess $f_0(\eta,\xi)$ to the unknown solution $f(\eta,\xi)$. Expending $\Phi(\eta,\xi;q)$ in Taylors series with respect to q, we have

$$\Phi(\eta,\xi;q) = \Phi(\eta,\xi,0) + \sum_{m=1}^{+\infty} f_m(\eta,\xi) q^m \ , \tag{21}$$

where

$$f_m(\eta,\xi) = \frac{1}{m!} \left. \frac{\partial^m \Phi(\eta,\xi;q)}{\partial q^m} \right|_{q=0} . \tag{22}$$

Note that Eq. (18) contain the auxiliary parameter $\hbar_f$. Assuming that $\hbar_f$ is correctly chosen so that the above series is convergent at $q = 1$, we have, using (20b), a solution series

$$f(\eta,\xi) = f_0(\eta,\xi) + \sum_{m=1}^{+\infty} f_m(\eta,\xi) \tag{23}$$

For simplicity, define

$$\mathbf{f}_m = \{f_0, f_1, f_2 \ldots, f_m\} \tag{24}$$

Differentiating the zero-order deformation equation (18) m times with respect to q, then setting $q = 0$, and finally dividing them by $m!$, we obtain a mth-order deformation equation

$$\mathcal{L}_f \left[f_m(\eta,\xi) - \chi_m f_{m-1}(\eta,\xi) \right] = \hbar_f R_m^f(\mathbf{f}_{m-1}) \ , \tag{25}$$

subject to the initial/boundary conditions

$$f_m(0,\xi) = \left. \frac{\partial f_m(\eta,\xi)}{\partial \eta} \right|_{\eta=0} = 0, \qquad \left. \frac{\partial f_m(\eta,\xi)}{\partial \eta} \right|_{\eta=\infty} = 0 \ , \tag{26}$$

where

$$R_m^f = \frac{1}{(m-1)!} \left. \frac{\partial^{m-1} \mathcal{N}\left[\Phi(\eta,\xi;q)\right]}{\partial q^{m-1}} \right|_{q=0} . \tag{27}$$

and

$$\chi_m = \begin{cases} 0, \ m = 1 \\ 1, \ m > 1. \end{cases} \tag{28}$$

Note that R_m^f is dependent on the integer power-law index κ. When $\kappa = 1$, we have

$$\begin{aligned} R_m^f(\mathbf{f}_{m-1}) &= \\ &= (1-\xi)\left(\frac{1}{2}\eta \frac{\partial^2 f_{m-1}}{\partial \eta^2} - \xi \frac{\partial^2 f_{m-1}}{\partial \eta \partial \xi} \right) + \frac{\partial^3 f_{m-1}}{\partial \eta^3} + \\ &+ \xi \left[\sum_{i=0}^{m-1} \left(f_i \frac{\partial^2 f_{m-1-i}}{\partial \eta^2} - \frac{\partial f_i}{\partial \eta} \frac{\partial f_{m-1-i}}{\partial \eta} \right) + 1 - \chi_m + M \left(\chi_m - \frac{\partial f_i}{\partial \eta} \right) \right] . \end{aligned} \tag{29}$$

When $\kappa = 2$, we have

$$\begin{aligned} R_m^f(\mathbf{f}_{m-1}) &= \\ &= (1-\xi)\left(\frac{1}{3}\eta \frac{\partial^2 f_{m-1}}{\partial \eta^2} - \xi \frac{\partial^2 f_{m-1}}{\partial \eta \partial \xi} \right) + 2A_{m-1} + \\ &+ \xi \left[\sum_{i=0}^{m-1} \left(\frac{4}{3} f_i \frac{\partial^2 f_{m-1-i}}{\partial \eta^2} - \frac{\partial f_i}{\partial \eta} \frac{\partial f_{m-1-i}}{\partial \eta} \right) + 1 - \chi_m + M \left(\chi_m - \frac{\partial f_i}{\partial \eta} \right) \right] , \end{aligned} \tag{30}$$

where

$$A_j = \sum_{i=0}^{j} \frac{\partial^2 f_i}{\partial \eta^2} \frac{\partial^3 f_{j-i}}{\partial \eta^3} \ . \tag{31}$$

When $\kappa = 3$, they read

$$\begin{aligned} R_m^f(\mathbf{f}_{m-1}) = \\ &= (1-\xi)\left(\frac{1}{4}\eta\frac{\partial^2 f_{m-1}}{\partial \eta^2} - \xi\frac{\partial^2 f_{m-1}}{\partial \eta \partial \xi}\right) + 3\sum_{i=0}^{m-1} A_i \frac{\partial^2 f_{m-i-1}}{\partial \eta^3} + \\ &+ \xi\left[\sum_{i=0}^{m-1}\left(\frac{3}{2} f_i \frac{\partial^2 f_{m-1-i}}{\partial \eta^2} - \frac{\partial f_i}{\partial \eta}\frac{\partial f_{m-1-i}}{\partial \eta}\right) + 1 - \chi_m + M\left(\chi_m - \frac{\partial f_i}{\partial \eta}\right)\right] . \end{aligned} \tag{32}$$

Let $f_m^*(\eta, \xi)$ denote a special solution of Eq. (25). According to (16), its general solution reads

$$f_m(\eta, \xi) = f_m^*(\eta, \xi) + C_1 \exp(-\eta) + C_2 \exp(\eta) + C_3 \ , \tag{33}$$

where the constants C_1, C_2, C_3 are determined by the boundary conditions (26), i.e.,

$$C_2 = 0, \qquad C_1 = \left.\frac{\partial f_m^*(\eta, \xi)}{\partial \eta}\right|_{\eta=0}, \qquad C_3 = -C_1 - f_m^*(0, \xi) \ . \tag{34}$$

In this way, it is easy to obtain the *linear* equation (25) one after the other in the order $m = 1, 2, 3, \ldots$, especially by means of the symbolic computation software, such as Mathematica, Maple.

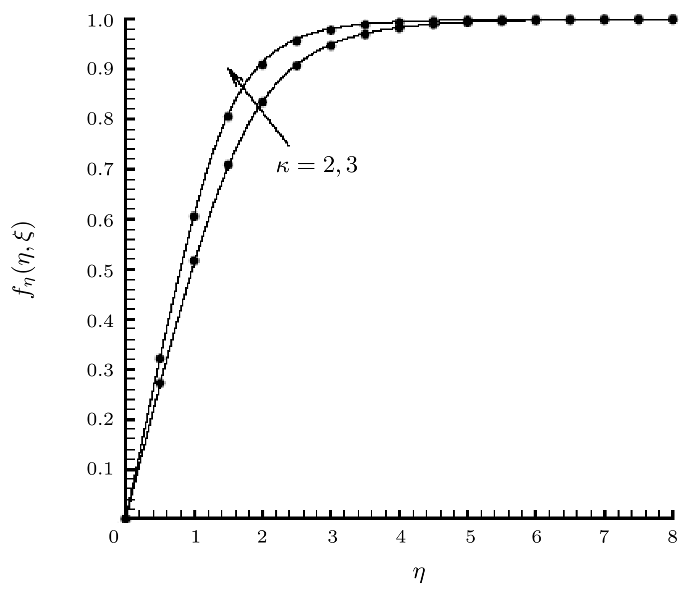

Fig. 1. Comparison of $f_\eta(\eta, \xi)$ of the analytic approximations with the numerical results at $\xi = 0$, when $\kappa = 2, 3$. Filled circles: numerical results; solid line: 25th-order HAM approximations.

Table 1. Comparison of $f_{\eta\eta}(0)$ for the steady-state ($\xi = 1$), when $\kappa = 1$.

M	25th-order Results	Pop's Results [33]	Sparrow *et al.* Results [34]	Numerical Results
0.5	1.41980	1.41975	1.418	1.41975
1	1.58536	1.58533	1.584	1.58534
0.5	1.73535	1.73536		1.73536
2	1.87353	1.87353	1.871	1.87353
3	2.12324			2.12324
4	2.34665		2.345	2.34665
5	2.55066			2.55066
10	3.39167			3.39167

3. Analysis of results. Liao [26] proved that as long as the solution series given by the homotopy analysis method converges, it must be one of exact solutions of the considered problem. Note that the solution series (23) contains the auxiliary parameter $\hbar_f$, which we can choose proper values by plotting the so-called $\hbar$-curves to ensure that the solution series (23) converges, as suggested by Liao [25].

When $\xi = 0$, corresponding to the initial state, our analytic series solutions agree well with the numerical results, as shown in Fig. 1. When $\xi = 1$, corresponding to the steady state, our analytic series solutions converge to the numerical ones in the whole region $0 \le \eta < +\infty$, as shown in Table 1 and Fig. 2. Similarly, in the whole region $\xi \in [0, 1]$, the series solutions (23) are convergent, as shown in Figs. 3–4. In fact, we find out that our series solutions (23) are convergent for

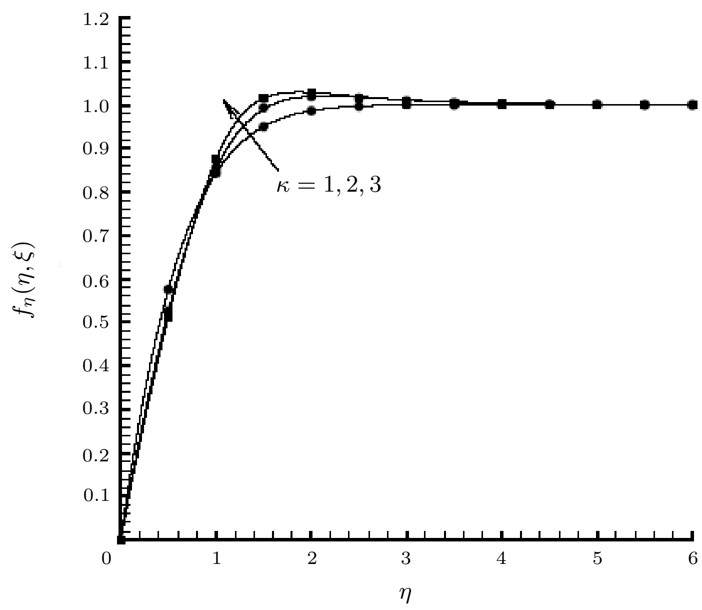

Fig. 2. Comparisons of $f_\eta(\eta, \xi)$ of the analytic approximations with the numerical results at $\xi = 1$ for the different power-law index κ, when $M = 1$. Filled circles: numerical results; solid line: 25th-order HAM approximations.

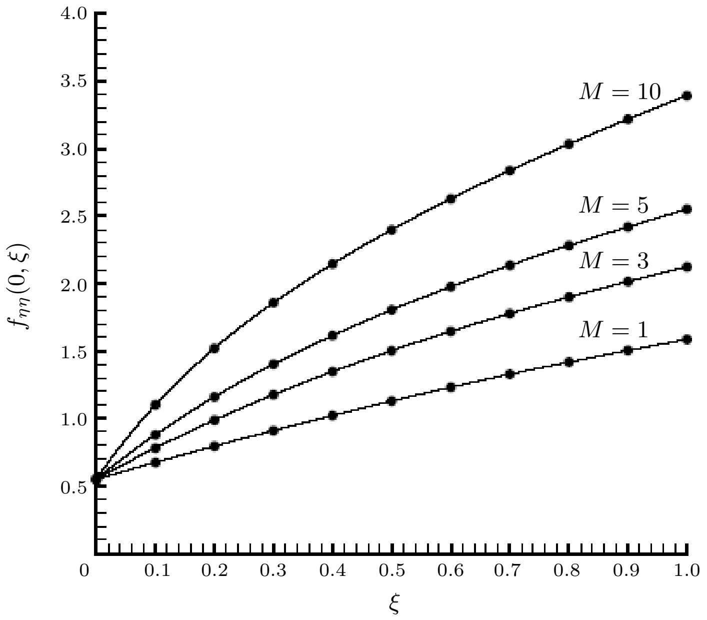

Fig. 3. Analytic approximations of $f_{\eta\eta}(0, \xi)$ for different M, when $\kappa = 1$. Solid line: 20th-order HAM approximations; filled circles: 25th-order HAM approximations.

all $\xi \in [0, 1]$. Thus, applying the homotopy analysis method, we obtain accurate analytic solutions uniformly valid for all $\xi \in [0, 1]$ in the whole region $0 \leq \eta < +\infty$.

The variation of the surface shear stresses with the dimensionless time ξ for several values of κ and M is illustrated in Figs. 3–4. At the start of the motion ($\xi = 0$), the surface shear stresses are independent on M, because M is multiplied

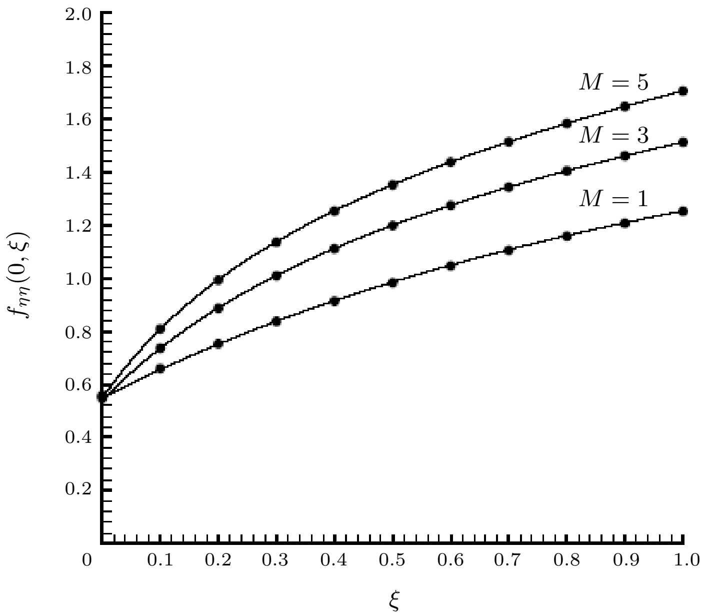

Fig. 4. Analytic approximations of $f_{\eta\eta}(0, \xi)$ for different M, when $\kappa = 2$. Solid line: 20th-order HAM approximations; filled circles: 25th-order HAM approximations.

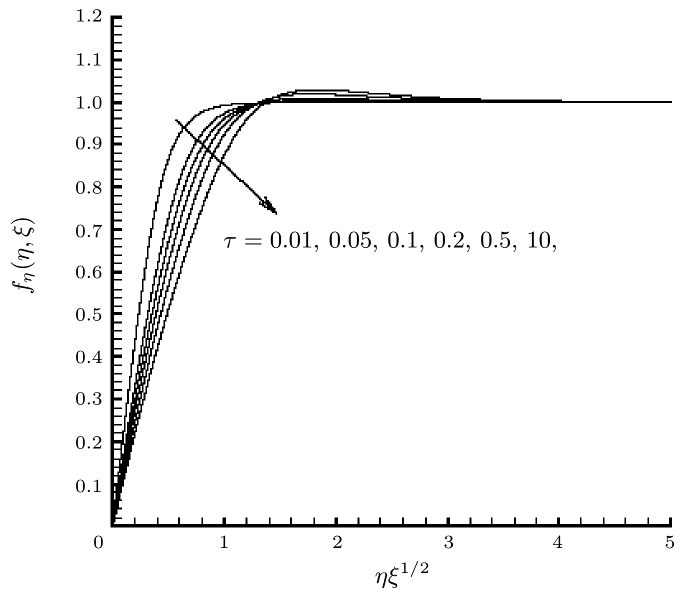

Fig. 5. The variation of the velocity profile $f_\eta(\eta, \xi)$, when $\kappa = 3$ and $M = 1$.

by ξ. Thus, M increases as ξ increases. The surface shear stresses increase as the magnetic parameter M increases because of the enhanced Lorentz force, which imparts additional momentum into the boundary layer. This reduces the boundary layer thickness.

The variation of the velocity profiles as a function of τ, when $\kappa = 3$ and $M = 1$, is shown in Fig. 5. We can see that these velocity profiles develop rapidly from rest as τ increases from zero to ∞.

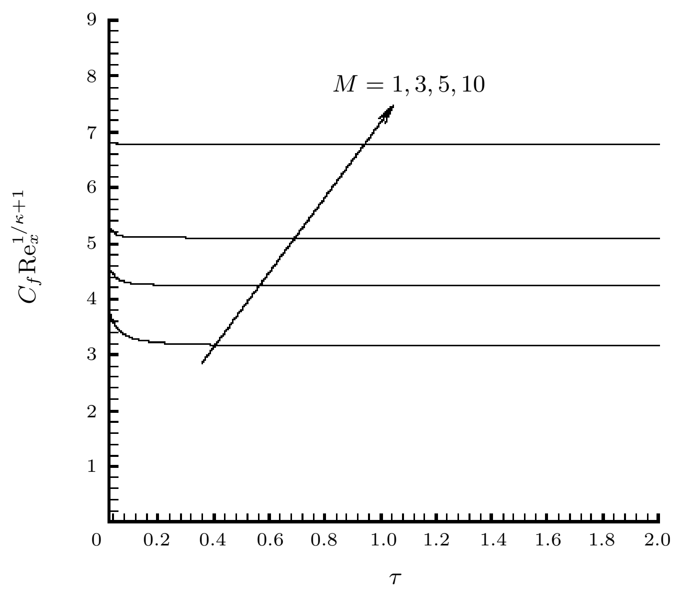

Fig. 6. The local skin friction coefficient as a function of τ for the different M when $\kappa = 1$.

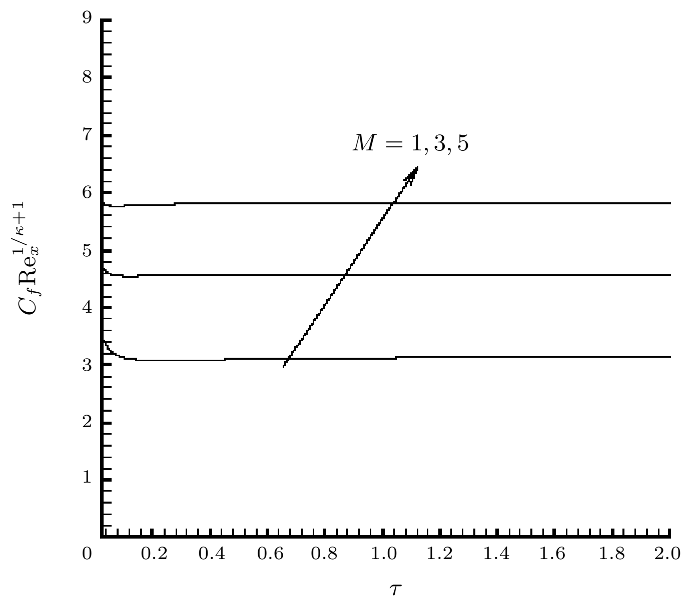

Fig. 7. The local skin friction coefficient as a function of τ for the different M when $\kappa = 2$.

The curves of the local skin friction coefficient C_f versus τ for a fixed value of either the power-law index n or the magnetic parameter M are shown in Figs. 6–8, respectively. Note that at the same dimensionless time $\tau \in (0, +\infty)$ and for the same power-law index κ the skin friction coefficient increases as the values of the magnetic parameter M enlarge.

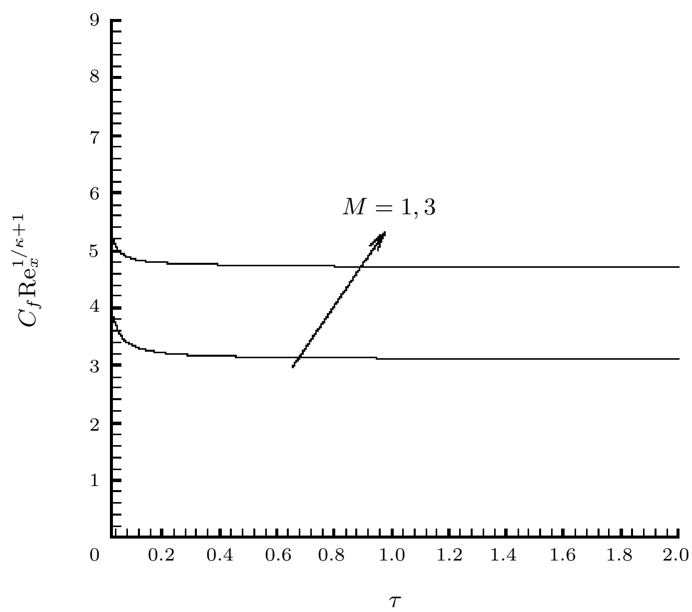

Fig. 8. The local skin friction coefficient as a function of τ for the different M when $\kappa = 3$.

Thus, by the homotopy method, we have obtained the analytic series solutions, which are accurate and uniformly valid for all dimensionless time $0 \leq \eta < \infty$ in the whole spatial region $0 \leq \eta < \infty$. To the best of our knowledge, such kind of analytic solutions have never been reported.

4. Conclusions. In this paper, we apply the homotopy analysis method to study the unsteady MHD flow of a non-Newtonian fluid near the forward stagnation point. By this analytic technique, the uniformly valid series solutions are obtained, which are valid for all time $0 \leq \tau < \infty$ and in the whole region $0 \leq \eta < \infty$. To the best of our knowledge, such kinds of analytic solutions have never been reported. The effects of integral power-law index of the non-Newtonian fluids for $\kappa = 1, 2, 3$ and magnetic parameter $M \leq 10$ on the velocity are considered. The proposed analytic approach has general meaning and thus may be applied similarly to other unsteady boundary-layer flows to get accurate analytic solutions valid for all the time.

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